

Expected Shortfall: a natural coherent alternative to Value at Risk

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Abstract

We discuss the coherence properties of Expected Shortfall (ES) as a financial risk measure. This statistic arises in a natural way from the estimation of the “average of the $100p\%$ worst losses” in a sample of returns to a portfolio. Here p is some fixed confidence level. We also compare several alternative representations of ES which turn out to be more appropriate for certain purposes.

KEY WORDS: Expected Shortfall; Risk measure; worst conditional expectation; tail conditional expectation; value-at-risk (VaR); conditional value-at-risk (CVaR); coherence; sub-additivity.

1 A four years impasse

Risk professionals have been looking for a coherent alternative to Value at Risk (VaR) for four years. Since the appearance, in 1997, of *Thinking Coherently* by Artzner et al [3] followed by *Coherent Measures of Risk* [4], it was clear to risk practitioners and researchers that the gap between market practice and theoretical progress had suddenly widened enormously. These papers in fact faced for the first time the problem of defining in a clearcut way what properties a statistic of a portfolio should have in order to be considered a sensible risk measure. The answer to this question was given through a complete characterization of such properties via an axiomatic formulation of the concept of *coherent risk measure*. With this result, risk management became all of a sudden a science in itself with its own rules correctly defined in a deductive framework. Surprisingly enough, however, VaR, the risk measure adopted as best practice by essentially all banks and regulators, happened to fail the exam for being admitted in this science. VaR is not a coherent risk measure because it simply doesn't fulfill one of the axioms of coherence.

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