

## COHERENT MEASURES OF RISK

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In this paper we study both market risks and nonmarket risks, without complete markets assumption, and discuss methods of measurement of these risks. We present and justify a set of four desirable properties for measures of risk, and call the measures satisfying these properties “coherent.” We examine the measures of risk provided and the related actions required by SPAN, by the SEC/NASD rules, and by quantile-based methods. We demonstrate the universality of scenario-based methods for providing coherent measures. We offer suggestions concerning the SEC method. We also suggest a method to repair the failure of subadditivity of quantile-based methods.

KEY WORDS: aggregation of risks, butterfly, capital requirement, coherent risk measure, concentration of risks, currency risk, decentralization, extremal events risk, insurance risk, margin requirement, market risk, mean excess function, measure of risk, model risk, net worth, quantile, risk-based capital, scenario, shortfall, subadditivity, tail value at risk, value at risk

### 1. INTRODUCTION

We provide in this paper a *definition* of risks (market risks as well as nonmarket risks) and present and justify a unified framework for the analysis, construction, and implementation of *measures* of risk. We do not assume completeness of markets. These measures of risk can be used as (extra) capital requirements to regulate the risk assumed by market participants, traders, and insurance underwriters, as well as to allocate existing capital.

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