Systemic Risk in a Unifying Framework for Cascading Processes on Networks

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Abstract

We introduce a general framework for models of cascade and contagion processes on networks, to identify their commonalities and differences. In particular, models of social and financial cascades, as well as the fiber bundle model, the voter model, and models of epidemic spreading are recovered as special cases. To unify their description, we define the net fragility of a node, which is the difference between its fragility and the threshold that determines its failure. Nodes fail if their net fragility grows above zero and their failure increases the fragility of neighbouring nodes, thus possibly triggering a cascade. In this framework, we identify three classes depending on the way the fragility of a node is increased by the failure of a neighbour. At the microscopic level, we illustrate with specific examples how the failure spreading pattern varies with the node triggering the cascade, depending on its position in the network and its degree. At the macroscopic level, systemic risk is measured as the final fraction of failed nodes, X^*, and for each of the three classes we derive a recursive equation to compute its value. The phase diagram of X^* as a function of the initial conditions, thus allows for a prediction of the systemic risk as well as a comparison of the three different model classes. We could identify which model class lead to a first-order phase transition in systemic risk, i.e. situations where small changes in the initial conditions may lead to a global failure. Eventually, we generalize our framework to encompass stochastic contagion models. This indicates the potential for further generalizations.

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1 Introduction

After the spread of the financial crisis in 2008, the term 'systemic risk' could be well regarded as the buzzword of these years. Although there is no consensus on a formal definition of systemic risk, it usually denotes the risk that a whole system, consisting of many interacting agents, fails. These agents, in an economic context, could be firms, banks, funds, or other institutions. Only very recently, financial economics is accepting the idea that the relation between robustness of individual institutions and systemic risk is not necessarily straightforward [24]. The debate on systemic risk, how it originates and how it is affected by the structure of the networks of financial contracts among institutions worldwide, is only at the beginning [6, 22]. From the point of view