Analysis of Systemic Risk: A Vine Copula-based ARMA-GARCH Model

Kuan-Heng Chen and Khaldoun Khashanah

Abstract— **In this paper, a model for analyzing each U.S. Equity sector's risk contribution (VaR ratio), the ratio of the Value-at-Risk of a sector to the Value-at-Risk of the system (S&P 500 Index), with vine Copula-based ARMA-GARCH (1, 1) modeling is presented. Vine copula modeling not only has the advantage of extending to higher dimensions easily, but also provides a more flexible measure to capture an asymmetric dependence among assets. We investigate systemic risk in 10 S&P 500 sector indices in the U.S. stock market by forecasting one-day ahead VaR and one-day ahead VaR ratio during the 2008 financial subprime crisis. Our evidence reveals vine Copula-based ARMA-GARCH (1, 1) is the appropriate model to forecast and analyze systemic risk.**

Index Terms—**Copula, Time Series, GARCH, Systemic Risk, Value-at-Risk**

I. Introduction

The definition of systemic risk from the Report to G20 Finance Ministers and Governors agreed upon among the International Monetary Fund (IMF), Bank for International Settlements (BIS) and Financial Stability Board (FSB) [3] that is *"(i) caused by an impairment of all or parts of the financial system and (ii) has the potential to have serious negative consequences for the real economy"*. Furthermore, *"G-20 members consider an institution, market or instrument as systemic if its failure or malfunction causes widespread distress, either as a direct impact or as a trigger for broader contagion."* A common factor in the various definitions of systemic risk is that a trigger event causes a chain of bad economic consequences, referred to as a *"domino effect"*. Given the definition of systemic risk quoted above, measuring systemic risk is done by estimating the probability of failure of an institute that is the cause of distress for the financial system. Therefore, we only consider the Value-at-Risk ratio of a sector to the system (S&P 500 Index), which interprets the sector risk adds to the entire system.

Girardi and Ergun [10] modify the CoVaR methodology, proposed by Adrian and Brunnermeier [1], by using the dynamic conditional correlation GARCH, while Hakwa *et al.* [11] also modified the methodology based on copula modeling. We extend their concepts and present vine Copula-based ARMA-GARCH (1, 1) VaR measure into a high dimensional analysis in systemic risk.

Sklar [22] introduced the copula, which describes the dependence structure between variables. Patton [18] defined the conditional version of Sklar's theorem, which extends the copula applications to the time series analysis. Otani and Imai [17] presented a basket CDSs pricing model with nested Archimedean copulas. However, multivariate Archimedean copulas are limited in that there are only one or two parameters to capture the dependence structure. Joe [12] introduced a construction of multivariate distribution based on pair-copula construction (PCC), while Aas *et al.* were the first to recognize that the pair-copula construction (PCC) principal can be used with arbitrary pair-copulas, referred to as the graphical structure of R-vines [14]. Furthermore, Dissmann *et al.* [7] developed an automated algorithm of jointly searching for an appropriate R-vines tree structures, the pair-copula families and their parameters. Accordingly, a high dimensional joint distribution can be decomposed to bivariate and conditional bivariate copulas arranged together according to the graphical structure of a regular vine. Besides, Rockinger and Jondeau [19] was the first to introduce the copula-based GARCH modeling. Afterwards, Lee and Long [15] concluded that copula-based GARCH models outperform the dynamic conditional correlation model, the varying correlation model and the BEKK model. In addition, Fang *et al.* [8] investigated that using Akaike Information Criterion (AIC) as a tool for choosing copula from a couple of candidates is more efficient and accurate than the multiplier goodness-of-fit test method.

The purpose of this paper is to present an application of the estimation of systemic risk in terms of the VaR/ES ratio by using vine copula-based ARMA-GARCH (1, 1) model, and the result provides the important conclusion that the method is a real-time and efficient tool to analyze systemic risk.

This paper has four sections. The first section briefly introduces existing research regarding systemic risk. The second section describes the definition of the VaR/ES ratio, and outlines the methodology of vine Copula-based GARCH (1, 1) modeling. The third section describes the data and explains the empirical results of VaR/ES ratio. The fourth section concludes our findings.

II. Methodology

A. Risk Methodology

The definition of Value-at-Risk (VaR) is that the maximum loss at most is $(1-\alpha)$ probability given by a period [20].

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