"The Co-CoVaR and some other Fair Systemic Risk Measures with Model Risk Corrections"☆

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PRELIMINARY VERSION

Abstract

This paper first highlights how fragile are the main measures of systemic risk of financial institutions and, secondly, proposes a simple correction to address this drawback linked to model errors of such measures. We show that main quantitative systemic risk measures are indeed very linked to extreme quantiles and that the ranking in terms of systemic risk importance highly depends upon the adopted measure, and, finally, that systemic risk measures are very sensitive to measurement errors. Following the approach of Boucher *et al.* (2013), we then propose a model risk correction to stabilize and reconciliate main systemic risk firm rankings. Our results finally suggest that the riskiness of risk measures is crucial for the sake of the fairness and the global financial system stability.

Keywords: Systemic Risk Measures, Model Risk, Expected Shortfall, Value-at-Risk.
J.E.L. Classification: C31, C52, G32.

1. Introduction

Following the experience of the recent 2007-2009 financial crisis, a special attention has been paid to the "macroprudential" regulation, *i.e.* the prevention of a financial system-wide

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distress that can adversely impacts the real economy. Identifying Systemically Important Financial Institutions (SIFI) constitutes a major concern for both academics and regulators. While historically, "systemic importance" has been associated with institution's size through the too-big-to-fail issue, recent events suggest a more complex picture. The interconnectedness of a SIFI is also determined by its interbank market linkages, and its effects are amplified by high leverage. The Financial Stability Board (2011) thus defines SIFI as "financial institutions whose distress or disorderly failure, because of their size, complexity and systemic interconnectedness, would cause a significant disruption to the wider financial system and economic activity". Regulators and policy makers called for a tighter supervision, extra capital requirements, and some liquidity buffers for SIFI (Financial Stability Board, 2011).

Measuring the financial systemic importance of financial institutions is thus crucial to identify SIFI but also since a Pigouvian tax calibrated on this systemic importance of institutions has been proposed to address the externalities of these too-big-to-fail or too-interconnected-to-fail institutions. Indeed, while all firms share the benefits of financial stability, market mechanisms do not exist to enforce firms to internalize the full cost of threats to stability created by their own activity.

As a consequence, several empirical measures have been proposed to give a consolidated view of a firm's systemic importance (see Bisias *et al.*, 2012; Castro and Ferrari, 2012)[1]. The main one relies on publicly available market data on the studied firms, which are by nature forward-looking, such as size, leverage, liquidity, but also complexity, and substitutability of a specific institution. It leads to the use of systemic risk measures such as, for the main ones, the Delta Conditional Value-at-Risk (ΔCoVaR) of Adrian and Brunnermeier (2011), the Multi-Conditional-VaR (MCoVaR) by Cao (2012), the Marginal Expected Shortfall (MES) of Acharya *et al.* (2010) and Brownlees and Engle (2012), the Component Expected Shortfall (CES) of Banulescu and Dumitrescu (2012) and the Systemic RISK measure (SRISK) of Acharya *et al.* (2012) and Brownlees and Engle (2012).

As the first historical measure, the CoVaR corresponds to the Value-at-Risk (VaR) of the financial system condition on a specific event affecting a given firm. The contribution of a firm to systemic risk (ΔCoVaR) may be intuitively explained as the difference between its CoVaR when the firm is in financial distress and the one when it is not. Second, the MES corresponds to a firm's expected equity loss when market falls below a certain threshold over a given horizon, namely a 2% market drop over 1 day for the Short-Run MES, and a 40% market drop over six months for the Long-Run MES (LRMES). The basic idea is that the banks with the highest MES contribute the most to market declines; thus, these banks may be considered as the greatest drivers of systemic risk. Third, the CES quantifies each firm's contribution to the overall risk adding the capital weight into the analysis and considering the system's risk is measured by Expected Shortfall (ES) of the market return. Finally, the SRISK measures the expected capital shortfall of an institution conditional on a crisis, using both the size of the firm but also its leverage. The intuition is that a firm with

[1] Bisias *et al.* (2012) for instance survey 31 measures of systemic risk in the economic and financial literature.

a largest potential capital loss, that may precisely occur during the system crisis, should be considered in the list of the most systemically risky institutions.

Over the recent years, numerous research papers have discussed these systemic risk measures[2], which, for some them, are now available on line[3]. These several recent contributions show that: 1) main systemic risk measures may be misleading in some cases (Löffler and Raupach, 2011; Daníelsson *et al.*, 2011); 2) definitions of risk measures are not neutral when measuring the impact of an institution on the global system (Gouriéroux and Monfort, 2013; Benoit *et al.*, 2013; Banulescu and Dumitrescu, 2012); 3) some systemic risk measures are very close to traditional quantile, quantile-related and co-volatility measures (Daníelsson *et al.*, 2011; Benoit *et al.*, 2013); 4) main systemic risk measures essentially rely on the accuracy of special extreme quantiles of the future return distribution (Daníelsson *et al.*, 2011; Benoit *et al.*, 2013). These stylized facts are also to be put in relation with a fifth one: the magnitude of model risk is largely under-estimated when computing VaR and other quantile-related quantities (Boucher *et al.*, 2013) and a fair system of systemic risk ranking is barely grounded (Hurlin *et al.*, 2012).

In the vein of Daníelsson *et al.* (2011) and Löffler and Raupach (2011), the goal of our article is to provide an assessment of the robustness and informativeness of main systemic risk measures in presence of model errors. As in Daníelsson *et al.* (2011) and Benoit *et al.* (2013), we thus first propose a comparison of the major systemic risk measures (CoVaR, ΔCoVaR, MES, CES, SRISK). Then we show, extending some of the results by Benoit *et al.* (2013) in a realistic setting that main systemic risk measures (namely ΔCoVaR and MES) are essentially linked to some quantile measures. After recalling the main qualities we can expect from model of quantiles, we then proposed model-risk corrected versions of main systemic risk measures. We hereafter confirm that 1) main risk measures essentially rely on our capacities to properly estimate direct quantile of firm and market return; 2) in terms of systemic risk impact of financial institutions is highly dependent of the adopted measure; and 3) we show that they are very sensitive of errors in the measure of extreme quantiles. We then use the approach of Boucher *et al.* (2013) in order to propose a correction that stabilizes systemic rankings, avoiding them to be merely arbitrary and random (as we claim they are at this stage). We thus start studying the case of the CoVaR (Adrian and Brunnermeier, 2011), and its corrected version (called Co-CoVaR), before generalizing to other main systemic risk measures such as MES, CES and SRISK. We finally show that suggested corrections are of importance and that the highlighted SIFI are different from those identified when using the standard non-corrected risk measures. In other words, we conclude that model risk in the computation of risk measures should be considered for the sake of the fairness and global financial system stability.

[2]Other related papers include Elsinger *et al.* (2006), Huang *et al.* (2009), Manganelli *et al.* (2010), Drehmann and Tarashev (2011), Kritzman *et al.* (2011), Billio *et al.* (2012), Giglio (2012), Acharya and Steffen (2013), Gauthier *et al.* (2012) and Gouriéroux and Monfort (2013).

[3] See http://vlab.stern.nyu.edu/welcome/risk/ (New York University), http://www.rmi.nus.edu.sg/ (Singapore) and http://www.crml.ch (Lausanne University) for online computations of systemic risk measures. See also Bisias *et al.* (2012) for some available systemic risk measure MatLab codes.

This article is organized as follows. We briefly introduce in the first section the main systemic risk measures. Secondly, we show in a traditional setting that main systemic risk measures can be expressed as function of quantiles. Thirdly, we present some of the most used tests realized on quantile. We suggest a model risk correction in the fourth section and illustrate the underlying idea on some VaR computations. The fifth section is devoted to an empirical application on several systemic risk measures in order to highlight the added value of such a correction. Last section finally concludes, whilst the appendix is dedicated to some definitions and extra results.

2. A Brief Description of the Main Systemic Risk Measures

In this section, we thus provide a brief formal definition for the main systemic risk measures. We consider I firms (with $i = [1, ...I]$), and denote r_{it} the (log-)return of a firm i at time t. Similarly, the market (log-)return r_{mt} is (approximately) the value-weighted average of all studied firm (log-)returns.[4]

The VaR measure corresponds to the worst potential loss of an asset i, at a given date, for some defined frequency and horizon, written for a probability threshold α such as:

$$\text{Prob}\left[r_{it} \leq VaR_{it}\left(1-\alpha\right)\right] = \alpha, \quad (1)$$

where Prob(.) is the unconditional probability (in the general case), r_{it} is the asset return at time t, α is a probability threshold such as $\alpha = F^{-1}\left[F\left(\alpha\right)\right]$ where $F^{-1}\left(.\right)$ is the inverse of the Cumulated Distribution Function.

The larger the value of VaR, the worse might be the loss.

A first related measure is the CoVaR (Adrian and Brunnermeier, 2011) that represents the quantile of market returns conditionally on some event observed for a unique financial institution i such as[5]:

$$\text{Prob}\left[r_{mt} \leq CoVaR_{mt}\left(1-\alpha\right) \mid r_{it} \leq VaR_{it}\left(1-\alpha'\right)\right] = \alpha, \quad (2)$$

where α' is another given (lower) probability level reflecting a major event on the firm i.

[4] We use here the original definition of the main measures, event if, because of the Jensen Inequality, the market log-return is generally higher that the value-weighted firm log-return, especially when we deal with extreme returns (*i.e.* far away from zero).

[5] In fact, there are two alternative definitions of CoVaR in the current literature. The original definition is in Adrian and Brunnermeier (2011) and a modified one is used, for instance, by Bernard *et al.* (2013), Girardi and Ergün (2012) and Mainik and Schaanning (2012). This modification was proposed by Girardi and Ergün (2012) to improve the compatibility of CoVaR with non-parametric estimation methods. In this study, we use this second generalized definition of CoVaR, assuming that the conditioning financial distress event refers to the return of institution j being at most at its VaR as opposed to being exactly at its VaR as in the original version. This change considers more severe distress events of institution j that are farther in the tail, does not produce some counter-intuitive effects and also improves the consistency of CoVaR with respect to the dependence parameter since, in this case, as shown by Mainik and Schaanning (2012), the CoVaR has a monotonic relation with the dependence parameter.

A direct extension presented Cao (2012), called Multi-CoVaR, propose to compute a CoVaR measure by considering simultaneously I financial institutions instead of only one, such as $(i = [1, ..., I])$:

$$\text{Prob}\left[r_{mt} \leq MCoVaR_{mt}\left(1-\alpha\right) \mid r_{1t} \leq VaR_{1t}\left(1-\alpha'\right), ..., r_{It} \leq VaR_{It}\left(1-\alpha'\right)\right] = \alpha. \quad (3)$$

This extension allows us to evaluate the quantile of market returns conditionally on some event observed for several institutions at the same time.

A close derived systemic risk measure of the CoVaR is the ΔCoVaR also proposed by Adrian and Brunnermeier (2011). The ΔCoVaR of a firm i is defined as the difference between the VaR of the financial system conditional on this particular firm, being in financial distress (extreme quantile), and the VaR of the financial system conditional on a firm i, being in its median state (median quantile). To define the distress of a financial institution, various definitions can be considered. Because they use a quantile regression approach, Adrian and Brunnermeier (2011) consider a situation in which the loss is precisely equal to its VaR:

$$\Delta CoVaR_{mt}\left(1-\alpha\right) = CoVaR_{mt}\left(1-\alpha\right) - CoVaR_{mt}\left(50\%\right). \quad (4)$$

The accuracy of estimations of the ΔCoVaR also directly depends on those of the different conditional quantiles used in the computation.

The Marginal Expected Shortfall (MES) of Acharya *et al.* (2010) and its conditional version in Brownless and Engle (2012), is the marginal contribution of a firm i to systemic risk, as measured by the Expected Shortfall (ES) of the system.[6] By definition, the ES at the (1-α%) level is the expected return in the worst α% of the cases, but it can be extended to the general case, in which the returns exceed a given threshold. Formally, the conditional ES of the system is defined as:

$$ES_{mt}(1-\alpha) = E\left[r_{mt} \left| r_{mt} \leq VaR_{mt}(1-\alpha)\right.\right]. \quad (5)$$

The MES thus corresponds to the partial derivative of the system ES with respect to the weight of a firm i in the economy such as:

$$MES_{it}(1-\alpha) = E\left[r_{it} \left| r_{mt} \leq VaR_{mt}(1-\alpha)\right.\right]. \quad (6)$$

The MES can be viewed as a natural extension of the concept of marginal VaR proposed by Jorion (2007) to the ES, which is a coherent risk measure (see Artzner *et al.*, 1999). It measures the increase in the risk of the system (measured by the ES) induced by a marginal increase in the weight of firm i in the system. The higher the firm MES, the higher the individual contribution of the firm to the risk of the financial system.

[6]Please note that the MES definition was already present in Jorion (2000, revised in 2007) and Scaillet (2004), and applied to systemic risk in Lehar (2005) for instance.

When we now compare the MES measure of Acharya *et al.* (2010) and Brownlees and Engle (2012), with the CoVaR by Adrian and Brunnermeier (2011), there is a difference in terms of the conditioning event and the direction; while MES looks at the returns of an institution when the financial system is in distress and experiencing losses, CoVaR does the opposite and looks at the returns of the financial system when an institution is in financial distress. This difference does not come from some intrinsic properties of the two measures, but is more linked to the usage that has been done for each. In fact, it is possible and straightforward to reverse the analysis for both measures. In that case CoVaR would correspond to the VaR of an institution conditional on the financial system being in distress, i.e., with a loss being below its VaR. This reverse CoVaR would be more in the spirit of MES as it would be measuring the exposure of an institution to the distress of the financial system. For the sake of simplicity, we will hereafter keep definitions of both measures as they were at origin.

A second derived systemic risk measure based on the ES of the market is the Component Expected Shortfall (CES) of Banulescu and Dumitrescu (2012) that measures the absolute contribution of a firm to the risk of the financial system (as opposed to the marginal contribution). It is obtained by calibrating the first derivative of the ES using weights w_{it} defined for each financial institution, such as:

$$CES_{it}\left[VaR_{it}(1-\alpha)\right] = w_{it}E\left[r_{it}\left|r_{mt} \leq VaR_{mt}(1-\alpha)\right.\right]. \qquad (7)$$

We hence assess the systemically riskiness of financial institutions at a given date t by measuring each firm's contribution to the financial system's expected loss measured by ES. The larger the contribution, the more systemically important the market return. In the literature on portfolio risk management, marginal and component risk measures are traditionally distinguished. While the Marginal VaR measures the effect of one unit change in the position of a given component on portfolio risk (measured by VaR), the Component VaR indicates how the portfolio VaR would change approximately if the component was deleted from the portfolio. The CES systemic risk measure is thus defined as a natural extension to the ES of the Component Value-at-Risk introduced by Jorion (2007).

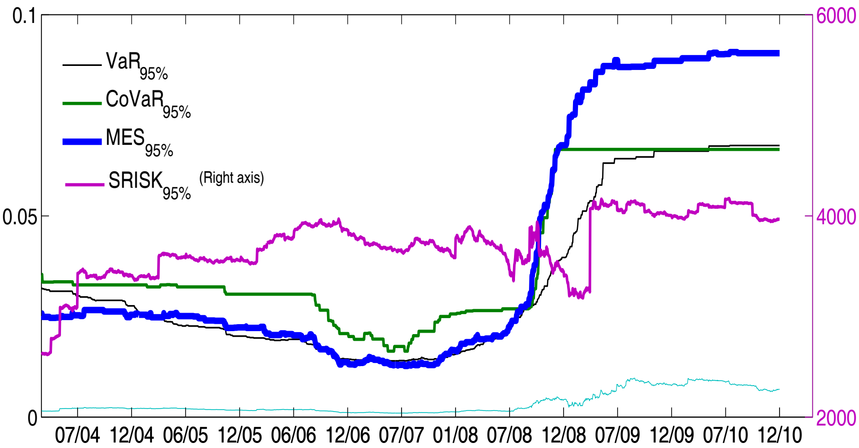
Lastly, the Systemic RISK measure (SRISK), proposed by Acharya *et al.* (2012) and Brownlees and Engle (2012), extends the MES in order to take into account both the liability and the size of financial institutions (see also Engle *et al.*, 2012). This measure corresponds to the expected capital shortfall of a given financial institution, conditional on a crisis affecting the whole financial system. In this perspective, the firms with the largest capital shortfall are assumed to be the greatest contributors to the crisis and are the institutions considered as the systemically riskiest. This systemic risk measure is defined such as:

$$SRISK_{it}(1-\alpha) = \max\left\{0, \gamma D_{it} - (1-\gamma)\, w_{it}\left[1 - LRMES_{it}(1-\alpha)\right]\right\}, \qquad (8)$$

where γ is the prudential capital ratio, D_{it} is the quarterly book value of total liabilities for the firm i, w_{it} is the daily market capitalization or market value of equity of the firm i, and $LRMES_{it}(1-\alpha)$ corresponds to the Long-Run Marginal Expected Shortfall, which is the expected equity loss of the firm i when market falls below 40% over six months (corresponding to some undefined quantile - see Acharya *et al.*, 2012, for details).

Note that the SRISK measure, which is positive by convention, is an increasing function of the liabilities and a decreasing function of the market capitalization. Then, the SRISK can be viewed as an implicit increasing function of the *quasi*-leverage defined by the ratio of the book value of total liabilities to the market capitalization. The SRISK also considers the interconnection of a firm with the rest of the system through the Long-Run MES. The latter corresponds to the expected drop in equity value the firm would experiment should the market falls by more than a given threshold within the next six months. Acharya *et al.* (2012) propose to approximate it using the daily MES (defined for a threshold equal to 2%) as $LRMES_{it}(1-\alpha) \simeq 1-\exp[18 \times MES_{it}(1-\alpha)]$. This approximation represents the firm expected loss over a six-month horizon, obtained conditionally on the market falling by more than 40% within the next six months.

Figure 1: VaR, CoVaR, MES, CES and SRISK Systemic Risk Measures for Bank of America



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3[rd] January, 2000 to the 31[st] December, 2010; computations by the authors. The figure shows the dynamic systemic risk measures (VaR, CoVaR, MES, CES and SRISK) at 95% confidence level for Bank of America.

For the sake of comparison, we will next use in our applications the exact same sample used in Acharya *et al.* (2010), Brownlees and Engle (2012), Benoit *et al.* (2013), and Banulescu and Dumitrescu (2012). It contains all the US financial institutions with a market capitalization superior to \$5 billions as of July 2007 (with a total of 94 firms). For our period of interest (from the 3[rd] January, 2000 to the 31[st] December, 2010), we compute daily (log-) returns on each asset of all the studied firms, and (log-) returns on the index of all firms weighted by their capitalization. Capitalization and daily closure prices are provided by CRSP, whilst quarter book values of the overall liability come from COMPUSTAT. The Appendix lists all the firms considered in our sample.

Figure 1 presents the various systemic risk measures on Bank of America (denoted BAC). As mentioned by Daníelsson *et al.* (2011), we clearly see that these various definitions essentially rely on some quantile-related weighted quantities as we will show more formally in the following section.

3. On Quantile Expressions of main Systemic Risk Measures

The systemic risk measures analyzed in this paper have been developed within different frameworks. For instance, Adrian and Brunnermeier (2011) and Engle *et al.* (2012) take into account tail dependences. Brownlees and Engle (2012) model time-varying linear dependencies and use a multivariate GARCH-DCC model to compute the MES, whilst Benoit *et al.* (2013) studies main systemic risk measures within an unified theoretical framework of the linear market model of Brownlees and Engle (2012).

Taking main elements of these previous seminal approaches (see also Löffler and Raupach, 2011), our system will be defined such as:

$$\begin{cases} r_{it} = r_f + \beta_{it} \times (r_{mt} - r_f) + \varepsilon_{it} \\ \qquad = \sigma_{it}\rho_{it}\varepsilon_{mt} + \sigma_{it}\sqrt{1-\rho_{it}^2}\varepsilon_{it} \\ r_{mt} = \sum w_{it} r_{it} \\ \epsilon_t \sim \mathbf{st}\left(\mathbf{0}, \sigma, \nu\right), \end{cases} \tag{9}$$

where $r_{it}, r_{mt}, r_f, \beta_{it}, \sigma_{it}, \sigma_{mt}, \rho_{it}, \varepsilon_{it}, \varepsilon_{mt}, w_{it}, \epsilon_t$, are, respectively, the return on firm i , the market return, the risk-free rate, the *beta* of firm i, the volatility of returns on firm i, the volatility of the market return, the linear Pearson correlation between return on firm i and the market return, the idiosyncratic return residual, the market return residual, the capital of firm i, the matrix of return residual (stacked by columns); $\mathbf{st}(.)$ represents a multi-variate Student's t-distribution with null means, volatilities of returns on firms and ν the vector of degrees of freedom for all assets.

First, in this setting, the return of firm i depends on the market return r_{mt} (through the CAPM relation between individual and the market return) and on an orthogonal firm-specific component ε_{it}. Secondly, the conditional standard deviations σ_{it} and σ_{mt}, and the conditional correlation ρ_{it} are time-varying. Thirdly, it is assumed here that processes governing the ϵ_t are *i.i.d.* over time and satisfies $E\left(\epsilon_t\right) = \mathbf{0}$ and $E\left(\epsilon_t\epsilon_t'\right) = \mathbf{I}_2$, the (2×2) Identity matrix. Fourthly, we here suppose that the multivariate distribution of the standardized return residuals is a Student, both for taking into account of fat-tail phenomenon and for the sake of being able to express analytically the link between VaR and ES. In this framework, the time-varying conditional correlations ρ_{it} fully captures the dependence between firm and market returns, which implies the (*a priori* unrealistic) assumption that standardized idiosyncratic return residual in ϵ_t are identically and independently distributed.

We presented hereafter some propositions derived in this general setting, as well as some direct illustrations of proposition statements estimated within the database presented above.

Proposition 1. *The MES of a given financial institution i is proportional to the ES of the market return, as well as an increasing function of the market VaR, and is equal to its own VaR at some other confidence level, such as:*

$$\begin{aligned} MES_{it}(\alpha) &= \beta_{it}\delta\left[VaR_{mt}(\alpha)\right] \\ &= VaR_{it}\left(\widetilde{\alpha}_{it}^{*}\right) \end{aligned} \tag{10}$$

where the proportionality coefficient β_{it} and function $\delta(.)$ are:

$$\begin{cases} \beta_{it} = cov\left(r_{it}, r_{mt}\right)/var\left(r_{mt}\right) \\ \quad = \rho_{it}\sigma_{it}/\sigma_{mt} \\ \delta\left(VaR_{mt}\right) = \frac{\nu+\left(VaR_{mt}\right)^{2}}{\nu-1}\frac{st\left(VaR_{mt}\right)}{1-\alpha}, \end{cases}$$

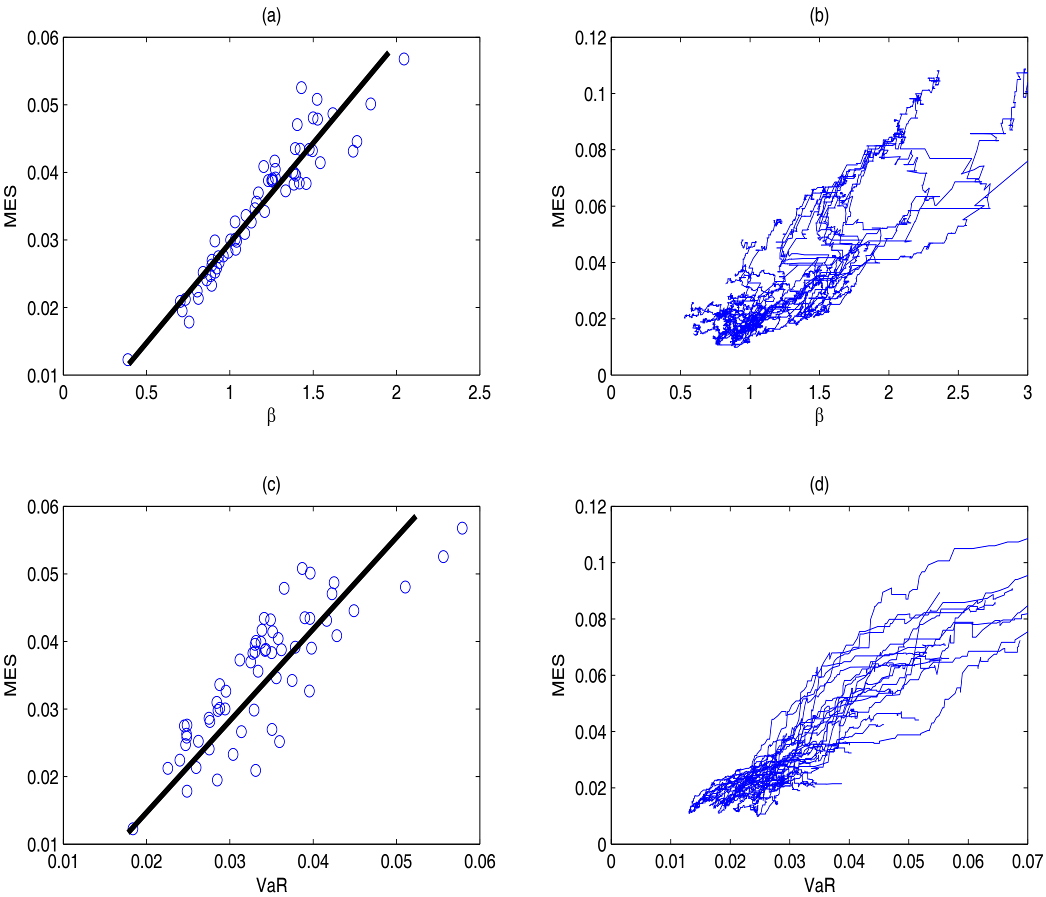
with β_{it} the time-varying beta of firm i, to $\delta(.)$ the intensification function linking the expected shortfall of the market, denoted $ES_{mt}(\alpha)$, and the Value-at-Risk of the market, denoted $VaR_{mt}(\alpha)$, with α and $\widetilde{\alpha}_{it}^{*}$ some arbitrary confidence levels and $st(.)$ the Student t-density function with ν degrees of freedom.

Proof. See Benoit *et al.* (2013), Andreev *et al.* (2005), Boucher *et al.* (2013) and Appendix.

In other words, we find here a similar result to the traditional CAPM (Sharpe, 1964) that says that the expected return is as a linear relation of the market *premium*, expressed here in terms of quantiles such as a specific quantile of a firm i can be expressed as the quantile of the market return (at the same confidence level) times the *beta*.

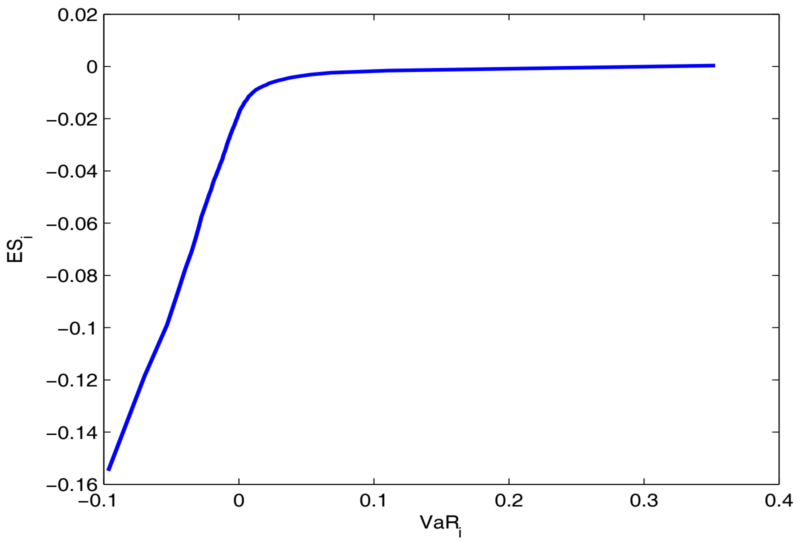
We here confirm the results by Benoit *et al.* (2013) that found this relation in the Brownless and Engle (2012) system framework (see Figure 2, top), as well as the result by Daníelsson *et al.* (2011) showing that main information in systemic risk measure is already contained in the VaR (see Figure 2, bottom).

Figure 2: Scatter Plot of MES95% *versus betas* and *versus* VaR99.5% of Financial Institutions



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors. The figure shows link between the MES95% estimated for each institution (y-axis) and its *beta* (x-axis, top) and its VaR99.5% (x-axis, bottom). Panel (a) and (c) refer to the the cross-sectional mean relation, and Panel (b) and (d) to the times series relation for each instituion. In the figure, circle represents a financial institution and the solid line is the OLS regression line.

Figure 3: Expected Shortfall Estimation for different VaR level of BAC



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors. The figure shows link between the ES (y-axis), and VaR (x-axis) for BAC estimated for each confidence level.

Proposition 2. *The $\Delta CoVaR$ of a given financial institution i is proportional to a measure of the dispersion of its return distribution (measured by a difference in VaR for different thresholds), as well as proportional to its own VaR at a different level, such as:*

$$\Delta CoVaR_{it}(\alpha) \quad = \quad \gamma_{it} VaR_{it}(\widetilde{\alpha}_{it}^{**}) = VaR_{it}(\widetilde{\alpha}_{it}^{***}), \tag{11}$$

where the proportionality coefficient is:

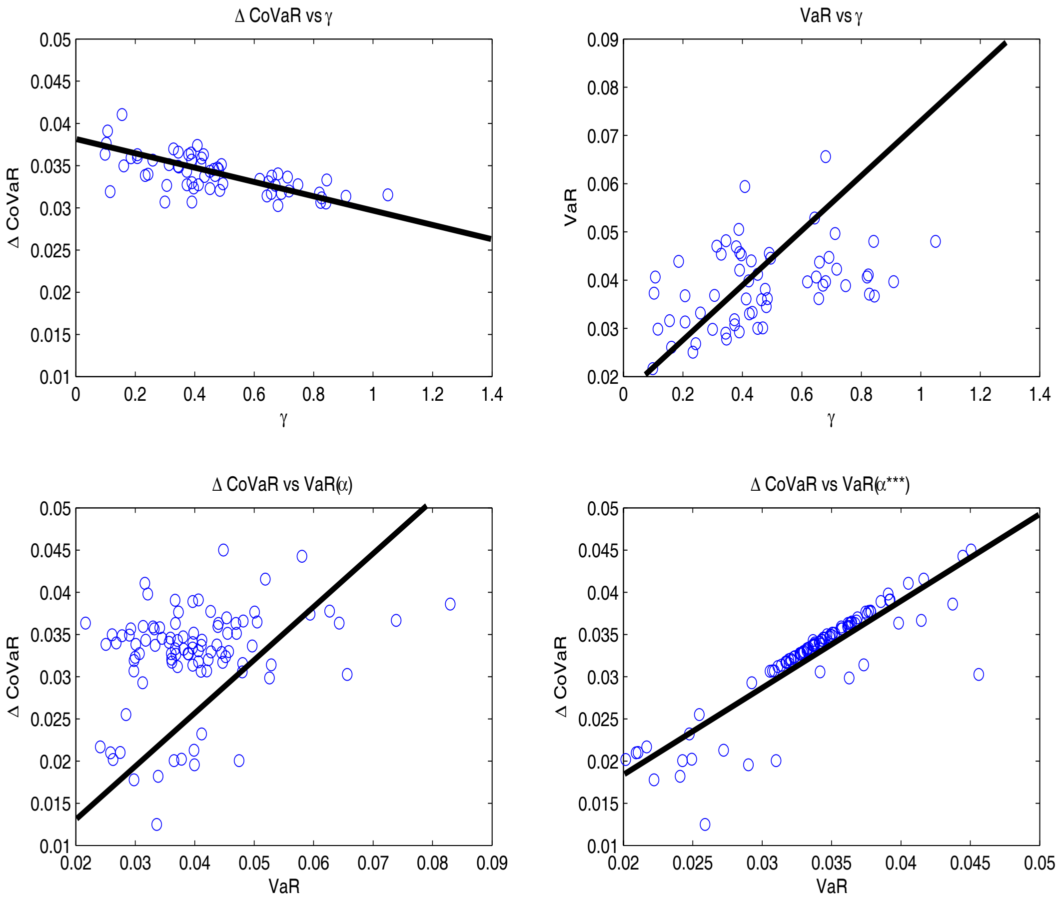
$$\begin{cases} \gamma_{it} = (\rho_{it})^2 \beta_{it} \\ \quad\; = \rho_{it}\sigma_{mt}/\sigma_{it}, \end{cases}$$

with γ_{it} the linear projection coefficient of the market return on the firm return.

Proof. See Benoit *et al.* (2013), Boucher *et al.* (2013) and Appendix.

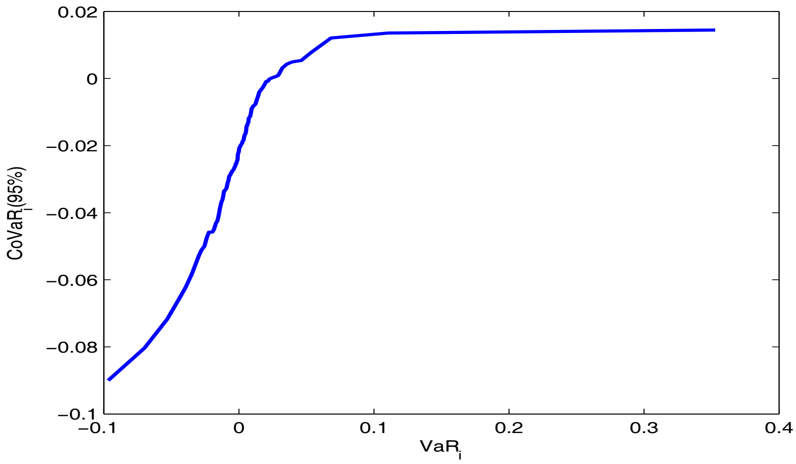
We see here a linear relation between the $\Delta CoVaR$ and the VaR of a firm, with a time-varying coefficient of the (cross-)relation that is linked to the correlation of return of the firm with the market return and the volatility of its return (see Figure 4).

Figure 4: Scatter Plot of $\Delta CoVaR95\%$ *versus* γ and *versus* $VaR_{it}(\tilde{\alpha}_{it}^{***})$ of Financial Institutions



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January 2000, to the 31^{st} December, 2010; computations by the authors. The figure shows the cross-sectional link between the the $\Delta CoVaR95\%$ estimated for each institution (y-axis) and its γ (x-axis, top) and its $VaR_{it}(\tilde{\alpha}_{it}^{***})$, with $\tilde{\alpha}_{it}^{***} \in [\bar{\tilde{\alpha}}_{it}^{***} - 5\%; \bar{\tilde{\alpha}}_{it}^{***} + 5\%]$ (x-axis, bottom). Each circle represents a financial institution and the solid line is the OLS regression line.

Figure 5: CoVaR 95% Estimation for different VaR level of BAC



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors. The figure shows link between the CoVaR (y-axis), and VaR (x-axis) for BAC estimated for each confidence level.

Proposition 3. *For a given financial institution i at time t, the ratio between its $\Delta CoVaR$ and its MES writes:*

$$\frac{\Delta CoVaR_{it}(\alpha)}{MES_{it}(\alpha)} = \frac{(\rho_{it})^2\left[VaR_{it}(\alpha) - VaR_{it}(.5)\right]}{\delta\left[VaR_{mt}(\alpha)\right]} = \frac{VaR_{it}(\widetilde{\alpha}_{it}^{***})}{VaR_{mt}(\widetilde{\alpha}_{it}^{****})}. \tag{12}$$

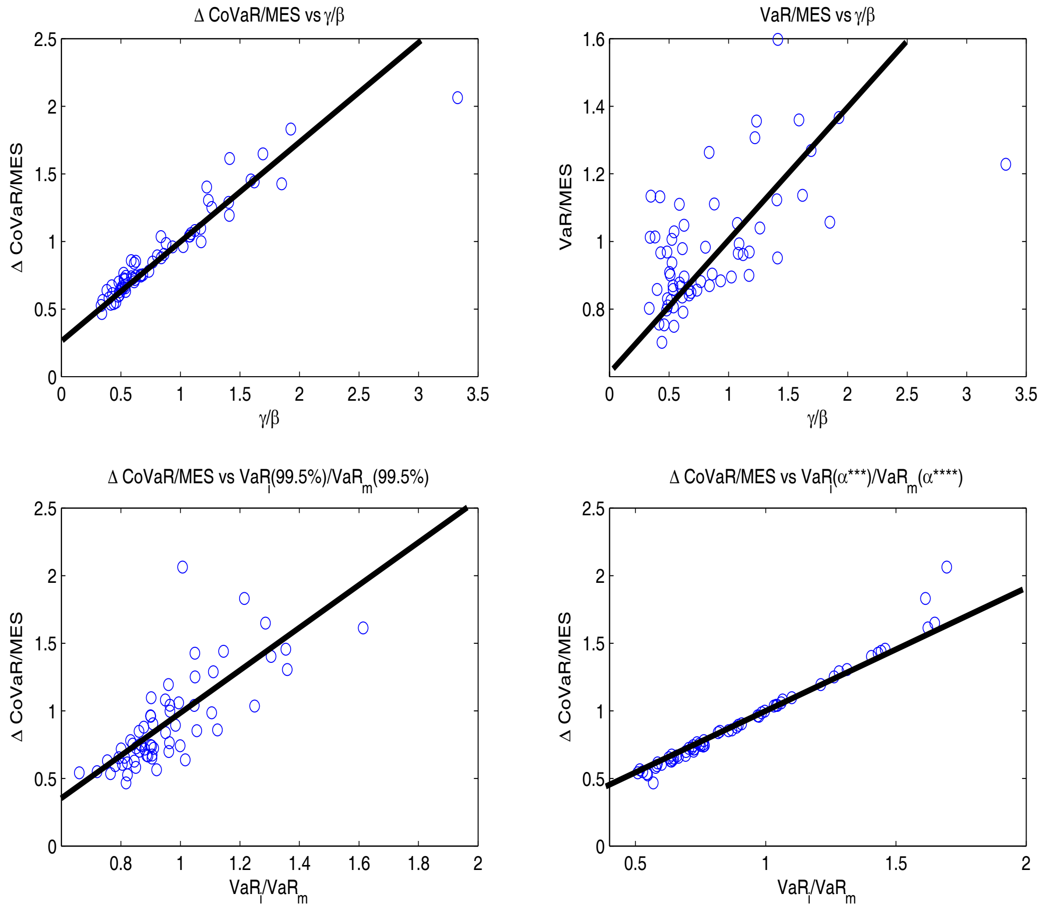
Proof. See Benoit *et al.* (2013), Boucher *et al.* (2013) and Appendix.

Figure 6 shows the cross-sectional link between the $\Delta CoVaR95\%/MES95\%$ ratios, estimated for each institution (y-axis) and theirs γ/δ ratios (x-axis, top) and theirs corresponding $VaR_{it}(\widetilde{\alpha}_{it}^{***})/VaR_{mt}(\widetilde{\alpha}_{it}^{****})$ ratios (x-axis, bottom).

As a result, the relation between $\Delta CoVaR$ and MES depends on a ratio of firm and market respective VaR.

As a preliminary conclusion, we may conclude that main systemic risk measures (namely $CoVaR$ and MES) essentially rely on the qualities of some VaR, whose accuracies are crucial for assessing the firm specific systemic risk component. We propose in the following section a short review of the main VaR tests, generally used in validation tests realized on quantile risk measures.

Figure 6: Scatter Plot of $\Delta CoVaR95\%/MES95\%$ *versus* γ/β and *versus* $VaR_{it}(\widetilde{\alpha}^{***}_{it})/VaR_{mt}(\widetilde{\alpha}^{****}_{it})$ of Financial Institutions



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3[rd] January, 2000 to the 31[st] December, 2010; computations by the authors. The figure shows the cross-sectional link between the $\Delta CoVaR95\%/MES95\%$ ratios, estimated for each institution (y-axis) and theirs γ/δ ratios (x-axis, top) and theirs $VaR_{it}(\bar{\alpha}^{***}_{it})/VaR_{mt}(\bar{\widetilde{\alpha}}^{****}_{it})$ ratios, with $\widetilde{\alpha}^{***}_{it} \in [\bar{\widetilde{\alpha}}^{***}_{it} - 5\%; \bar{\widetilde{\alpha}}^{***}_{it} + 5\%]$ and $\widetilde{\alpha}^{****}_{it} \in [\bar{\widetilde{\alpha}}^{****}_{it} - 5\%; \bar{\widetilde{\alpha}}^{****}_{it} + 5\%]$ (x-axis, bottom). Each circle represents a financial institution.

4. Some Common Value-at-Risk Tests

Since Basel II regulation, Value-at-Risk (VaR) of financial institutions are used to define the requirements in terms of capital, fixed in order to limit the effects of market risks. However, we have recently seen with the last financial crisis episodes that risk forecast methods proposed in the previous decades were rather inappropriate. In July 2009, the Basel Committee on Banking Supervision issued a directive (BCBS, 2009) finally requiring that financial institutions quantify model risk relying on some good properties of a VaR model.

A vast variety of tests are now proposed in the literature to gauge the accuracy of VaR estimates, mainly based on the three good properties that we should expect from risk models, such as the right frequency of violations, the independence of hits, and the reasonable magnitude of exceptions. The first test for a good VaR is the so-called "traffic light" approach used nowadays in the regulatory framework, which is related to the proportion of failure test. The Unconditional Coverage test (Kupiec, 1995) indeed attempts to determine whether the observed frequency of exceptions is consistent with the expected frequency of exceptions according to both a chosen VaR model and a confidence interval (an exception occurs when the *ex post* return is below of the *ex ante* VaR). The so-called "hit variable" associated to the *ex post* observation of violations of an estimated VaR (denoted $\mathrm{EVaR}(\cdot)$) at the threshold α and time t (denoted $I_t^{\mathrm{EVaR}}(\alpha)$), is defined as (with the previous notations):

$$I_t^{\mathrm{EVaR}(\cdot)}(\alpha) = \begin{cases} 1 \text{ if } r_t < -\mathrm{EVaR}(\hat{\theta}, \alpha)_{t-1} \\ 0 \text{ otherwise,} \end{cases} \tag{13}$$

where r_t is the return on portfolio P at time t, with $t = [1, 2, \dots, T]$.

If we assume that the $I_t^{\mathrm{EVaR}}(\cdot)$ variables are Independently and Identically Distributed, then, under the Unconditional Coverage hypothesis (Kupiec, 1995), the total number of VaR exceptions (Cumulated hits) follows a Binomial distribution (Christoffersen, 1998), denoted $B(T, \alpha)$, such as:

$$\mathrm{Hit}_t^{\mathrm{EVaR}(\cdot)}(\alpha) = \sum_{t=1}^{T} I_t^{\mathrm{EVaR}(\cdot)}(\alpha) \sim B(T, \alpha), \tag{14}$$

where the symbol $\sim$ refers to the convergence in distribution.

Under the null hypotesis of good VaR, the Likelihood Ratio $LRuc$ has an asymptotic distribution such as:

$$LRuc^{I_t^{\mathrm{VaR}(\cdot)}(\alpha)} = 2\{\log[\widehat{\alpha}^{T_I}(1-\widehat{\alpha}^{T-T_I})] - \log[\alpha^{T_I}(1-\alpha^{T-T_I})]\} \xrightarrow{d} \chi^2(1), \tag{15}$$

where the symbol $\xrightarrow{d}$ denotes the convergence in distribution of the test statistic, T is the number of observations, $T_I = T \times E\left[I_t^{\mathrm{EVaR}(\cdot)}\right]$ is the number of exceptions and $\widehat{\alpha} = T_I/T$ is the unconditional coverage.

A perfect sequence of (corrected) empirical VaR in the sense of this test (not too aggressive, but not too confident), is such that it respects condition (15).

The second test for a good VaR concerns the independence of forecasting errors. The independence hypothesis is associated to the idea that if the VaR model is correct then violations associated to VaR forecasting should be independently distributed, it is also called the independence of exceptions hypothesis. If the exceptions exhibit some type of "clustering", then the VaR model may fail to capture the profit and loss variability under certain conditions, which could represent a potential problem down the road. Christoffersen (1998) supposes that, under the alternative hypothesis of VaR inefficiency, the process of $I_t^{\text{EVaR}}(\alpha)$ violations is modelled with a Markov chain whose matrix of transition probabilities is defined by:

$$\Pi = \begin{pmatrix} \pi_{00} & \pi_{01} \\ \pi_{10} & \pi_{11} \end{pmatrix}, \tag{17}$$

where $\pi_{ij} = \text{Prob}\left[I_t^{\text{EVaR}}(\alpha) = j | I_{t-1}^{\text{EVaR}} = i\right]$.

This Markov chain reflects the existence of an order 1 memory in the process $I_t^{\text{EVaR}}(\alpha)$. The probability of having a violation (or not having one) for the current period depends on the occurrence (or not) of a violation (for the same level of coverage rate) in the previous period. Christoffersen (1998) shows that the likelihood ratio for the test is:

$$\text{LRind}^{I_t^{\text{EVaR}}(\alpha)} = 2\left[\log L^{I_t^{\text{EVaR}}(\alpha)}(\pi_{01}, \pi_{11}) - \log L^{I_t^{\text{EVaR}}(\alpha)}(\pi, \pi)\right] \xrightarrow{d} \chi^2(1), \tag{18}$$

where $L^{I_t^{\text{EVaR}}(\alpha)}(\pi_{01}, \pi_{11})$ is thus the likelihood under the hypothesis of the first-order Markov dependence, and $L^{I_t^{\text{EVaR}}(\alpha)}(\pi, \pi)$ is the likelihood under the hypothesis of independence $\pi_{01} = \pi_{11} = \pi$ such as:

$$L^{I_t^{\text{EVaR}}(\alpha)}(\pi_{01}, \pi_{11}) = (1 - \pi_{01})^{T_{00}} \pi_{01}^{T_{10}} (1 - \pi_{11})^{T_{10}} \pi_{11}^{T_{11}},$$

and:

$$L^{I_t^{\text{EVaR}}(\alpha)}(\pi, \pi) = (1 - \pi)^{T_{00}+T_{10}} \pi^{T_{01}+T_{11}},$$

with T_{ij} the number of observations in the state j for the current period and at state i for the previous period, $\pi_{01} = T_{01}/(T_{00} + T_{01})$, $\pi_{11} = T_{11}/(T_{10} + T_{11})$ and $\pi = (T_{01} + T_{11})/T$.

A perfect sequence of corrected (empirical) VaR in the sense of this test (*i.e.* not too reactive, but not too smooth), is such that it respects condition (18).

A third class of tests focuses on the magnitude of the losses experienced when VaR estimates are exceeded. The underlying idea is that a small violation might be acceptable but that a large one can lead to bankruptcy. In other words, not only the number of violations should be under scrutiny, but also super-exceptions (as defined in Colletaz *et al.*, 2012). Berkowitz (2001) proposes a hypothesis test for determining whether the magnitudes of observed VaR exceptions are consistent with the underlying VaR model. The key intuition is that VaR exceptions are treated as continuous random variables and not converted into the indicator variable used for the coverage tests. For this test, Berkowitz (2001) transforms the empirical series into a standard normal z_{t+1} series. If the observed quantile q_{t+1} with the distribution forecast f_{t+1} for the observed portfolio return r_t, is defined as:

$$q_{t+1} = \int_{-\infty}^{r_{t+1}} f_{t+1}(r)dr. \tag{19}$$

The z_{t+1} values are then compared to the normal random variables with the desired coverage level of the VaR estimates:

$$z_{t+1} = \Phi^{-1}(q_{t+1}), \tag{20}$$

where $\Phi^{-1}(\cdot)$ is the quantile function of the standard normal density.

If the VaR model generating the empirical quantiles is correct, then the γ_{t+1} series should be identically distributed with the unconditional mean and standard deviation, denoted (μ, σ) that should equal to $(0, 1)$, such as:

$$\gamma_{t+1} = \begin{cases} z_{t+1} \text{ if } z_{t+1} < \Phi^{-1}(\alpha) \\ 0 \text{ otherwise,} \end{cases} \tag{21}$$

where $\Phi(\cdot)$ is the standard normal Cumulative Distribution Function.

Finally, the corresponding test statistic is:

$$\text{LRmag}^{\gamma_{t+1}} = 2\left[L_{\text{mag}}^{\gamma_{t+1}}(\mu,\sigma) - L_{\text{mag}}^{\gamma_{t+1}}(0,1)\right] \xrightarrow{d} \chi^2(2), \tag{22}$$

where:

$$\begin{aligned} L_{\text{mag}}^{\gamma_{t+1}}(\mu,\sigma) = & \sum\nolimits_{\{\gamma_{t+1}=0\}} \log\left\{1 - \Phi\left\{\tfrac{\Phi^{-1}(\alpha)-\mu}{\sigma}\right\}\right\} \\ + & \sum\nolimits_{\{\gamma_{t+1}\neq 0\}} \left\{-\tfrac{1}{2}\log(2\pi\sigma^2) - \tfrac{(\gamma_{t+1}-\mu)^2}{2\sigma^2} - \log\left\{\Phi\left\{\tfrac{\Phi^{-1}(\alpha)-\mu}{\sigma}\right\}\right\}\right\}. \end{aligned}$$

A perfect sequence of (corrected) empirical VaR in the sense of this test (*i.e.* not too conservative, but not too over-confident), is such that it respects condition (22).

For both unconditional and conditional coverage tests[7], Escanciano and Olmo (2009, 2010 and 2011) alternatively approximate the critical values of these tests by using a sub-sampling bootstrap methodology, since they show that the coverage VaR backtest is affected by model misspecification. Thus, they propose to use robust sub-sampling techniques to approximate the true distribution of these tests. However, they also show that although the estimation risk can be diversified by choosing a large in-sample size relative to an out-of-sample one, the risk associated to the model cannot be eliminated using sub-sampling.

Indeed, let $G_x(x)$ denote the Cumulative Distribution Function of the test statistic k for any $x \in$ IR, and, $k_{b,t} = K(t, t+1, \cdots, t+b-1)$, with $t = [1, 2, \cdots, T-b+1]$, the test statistic computed with the subsample $[1, 2, \cdots, T-b+1]$ of size b.

[7]The conditional coverage test proposed by Christoffersen (1998) combines an unconditional coverage test (the frequency corresponds to the probability) and the independence test (see above).

Hence, the approximated sampling Cumulative Distribution Function of k, denoted $G_{k_b}(x)$, built using the distribution of the values of $k_{b,t}$ computed over the $(T-b+1)$ different consecutive subsamples of size b is given by:

$$G_{k_b}(x) = (T-b+1)^{-1} \sum_{t=1}^{T-b+1} 1\mathrm{I}_{\{k_{b,t}<x\}}. \tag{23}$$

The $(1-\tau)^{\text{th}}$ sample quantile of G_{k_b}, is given by:

$$c_{k_b,1-\tau} = \underbrace{\inf}_{x\in \mathrm{IR}} \{x \,|\, G_{k_b}(x) \geq 1-\tau\}\,. \tag{24}$$

Since we have now exposed main good qualities of VaR and the most common tests for them, we can suggest in the following a correction for model risk that we herein illustrate with VaR.

5. Model Risk and Corrections

According to Boucher *et al.* (2013) who propose some VaR corrections for model risk, we suggest hereafter a practical method to deal with model uncertainty that can also be applied in the framework of systemic risk measures. It makes uses of the past historical errors related to specific estimated models. While it is not possible to optimally adjust for biases (i.e. systematic repeated errors), we can approximate them by adjusting the VaR forecasts according to the historical performance of the same model. In other words, past errors are used to adjust future forecasts under the rule of thumb of the learning-by-failing principle. The procedure thus consists of adjusting the VaR estimates with the minimum correction needed, at any time, for passing some defined backtests corresponding the desirable properties of a model risk (as presented in the previous section).

In the following, we first briefly present the backtests considered in our empirical applications to calibrate the correction added to the estimated VaR. These backtests represent the main properties expected for VaR estimates: adequate frequency, independence and low magnitude of VaR errors. Then, we use our practical framework to compare risk models based on the calibration of *ex post* corrections, and we investigate the *ex ante* dynamics and the distribution of these corrections.

We first define the Imperfect Model Adjusted VaR (IMAVaR) as:

$$\mathrm{IMAVaR}(\hat{\theta}_1, \alpha, n) = \mathrm{EVaR}(\hat{\theta}_1, \alpha, n) + \mathrm{adj}(\theta_0, \theta_1, \hat{\theta}_1, \alpha, n), \tag{25}$$

where $\mathrm{EVaR}(\cdot)$ is an estimated VaR with a specific risk model, θ_0 the true parameters, $\hat{\theta}_1$ are model parameters estimated with T observations, α the level of confidence, n the number of assets, and $\mathrm{adj}(\theta_0, \theta_1, \hat{\theta}_1, \alpha, n)$ the minimum VaR adjustment for the risk model to be validated by the supervisors, such that:

$$\mathrm{IMAVaR}(\hat{\theta}_1, \alpha, n) = \underbrace{\sup}_{\mathrm{VaR}\,\in\,\mathrm{IR}} \{\mathrm{VaR}(\alpha, n)^*\}\,, \tag{26}$$

where $\text{VaR}(\cdot)^*$ is a set of VaR, from a model approved by the supervisor, and $\text{IMAVaR}(\cdot)$ is the limit highest VaR (the less conservative VaR) such that the supervisor still validate the model.

Generally speaking, the better the VaR model and the lower the minimum required adjustment and *vice-versa*. We now have to explicit the limit VaR that bounds the IMAVaR.

Under H_0 hypothesis a "good" VaR (*i.e.* a VaR sequence passes the tests), we have:

$$\begin{cases} LRuc^{I_t^{\text{VaR}(\cdot)}}(\alpha) \xrightarrow{d} \chi^2(1) \text{ for the "Hit" test} \\ \\ \text{LRind}_t^{I^{\text{VaR}(\cdot)^*}}(\alpha) \xrightarrow{d} \chi^2(1) \text{ for the "Independence" test} \\ \\ \text{LRmag}_t^{\gamma_{t+1}}(\alpha) \xrightarrow{d} \chi^2(2) \text{ for the "Exception Magnitude" test,} \end{cases} \tag{27}$$

where the symbol $\xrightarrow{d}$ means the limit distribution of the test statistic under the unconditional coverage hypothesis for the Kupiec (1995) test, an independence hypothesis for the independence test and a normal hypothesis of magnitude of exceptions for the magnitude test.

We now have to search for the minimal adjustment value q^* that allows us to pass all the tests (one-by-one or, alternatively, altogether). For a given VaR forecast and the bounding range for the tests above, we can obtain the $IMAVaR$ that respects condition (27)) or their sub-sampled versions. More precisely, Given a sequence of predictions $\left\{VaR_t(\hat{\theta},\alpha) : t = 1, \cdots, T\right\}$, we construct the set of values $q \in \mathbb{R}$ such that the sequence $\left\{VaR_t(\hat{\theta},\alpha) + q : t = 1, \cdots, T\right\}$ passes several backtests. Il we denote the set of accepted adjustments by $A_T(\alpha)$, the optimal adjustment is given by[8]:

$$q_T^* = \arg \underbrace{\min}_{q\in A_T(\alpha)} \{q\}. \tag{28}$$

We use a numerical optimisation technique to solve the program (28). During the adjustment process, we search for the optimal adjustment, starting with a large negative value of q^*, increasing it slowly, until the adjusted VaR allows us to pass all the tests[9].

The program (28) gives the optimal value of adjustment of the imperfect VaR estimation to become a good VaR. This means that the H_0 hypothesis (of a "good" VaR model) is

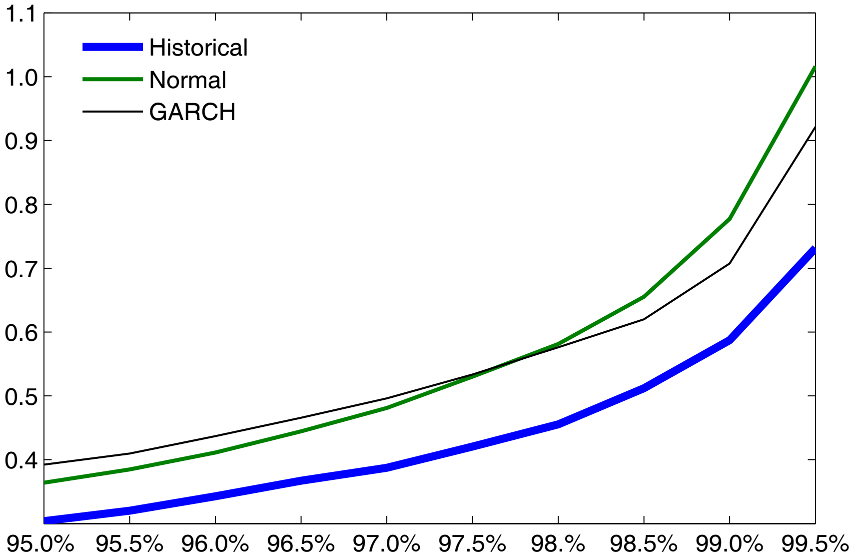
[8]On a theoretical basis, $A_T(\alpha)$ might, of course, be empty and the optimal adjustment can be positive. However, as the sample goes large, these two situations are very unlikely since negative error might soon appears (please, see Figure 6 below).

[9]for information, we here used a looped grid-search algorithm, adding successively a small increment on the top of the VaR (+.1% of the EVaR at each round), starting from large negative value and increasing until the test is finally passed.

true for the selected backtest method, so that the test statistic is lower than critical values for all tests at the threshold α. In the following, in order to distinguish the effect of each test, we will provide each correction separately corresponding to each of the tests taken alone.

As a first illustration, Figure 7 provides the minimum adjustments (errors), denoted $\underline{q}^*$ as solutions of the program (28). We first only considering the hit test, for the Historical, the Gaussian and the GARCH VaR computed for Bank of America with daily data from the 3^{rd} January, 2000 to the 31^{st} December, 2012. The figure represents the minimal adjustment (in percent of the underlying VaR) necessary to respect the hit ratio *criterion* according to the VaR level of confidence (95% to 99.5%). This minimal adjustment is here considered as a *proxy* of the economic value of the model risk; it is expressed as a proportion of the observed average VaR.

Figure 7: Minimum Model Risk Adjustment Factor for the Hit Test associated to Historical, Gaussian and GARCH VaR for Bank of America according to the Level of Confidence



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2012; computations by the authors. This figure represents on the y-axis the minimal adjustment (in percent of the underlying VaR) necessary to respect the hit ratio *criterion* according to the VaR level of confidence (x-axis). This minimal adjustment is here considered as a *proxy* of the economic value of the model risk; it is expressed as a proportion of the observed average VaR. The historical VaR is here computed on a weekly horizon as an empirical quantile using 5 years of past returns. The Gaussian and the GARCH VaR are here computed on a weekly horizon as a parametric quantile using 5 years of past returns to estimate the parameters.

In other words, we show the minimal constant that should be added to the quantile estimation for reaching a VaR sequence that passes the hit test at all times (here with full information at time T). We can see that the corrections range from (almost) 0 to 100% and increase with the quantile (estimated VaR are too aggressive), underlying that model risk is the largest for high quantiles. The comparison of the three methods is in favour of the GARCH method since the error is lower for all quantiles and that the difference between methods (with full information on the total sample) is quite similar and rather independent of the confidence level.

The following section now deals with the idea to apply the proposed correction each time we will need to compute a quantile that appears in the computation of systemic risk measures. We also illustrate in this section the impact of the suggested correction when estimating systemic risk for financial institutions.

6. Systemic Risk Measures with Model Risk

This section investigates the consequences of the model risk of systemic risk measures and proposes to apply the procedure described in the previous sections to these systemic risk measures. Our objective is to deliver systemic risk measures robust to the model risk.

6.1. The Adjusted Systemic Risk Measures

We present hereafter adjusted systemic risk measures. The main idea is to modify, in the formulation of the existing measure, the estimated VaR by the adjusted VaR.
A first adjusted systemic risk measure is the Co-CoVaR that represents the quantile of market returns conditionally on some event observed for a unique financial institution i such as (with the previous notations):

$$\text{Prob}\left[r_{mt} \leq \text{Co-CoVaR}_{mt}\left(1-\alpha^*\right) \mid r_{it} \leq VaR_{it}\left(1-\alpha'\right)\right] = \alpha^*, \tag{29}$$

where $\text{Co-CoVaR}_{mt}\left(1-\alpha^*\right)$ is adjusted CoVaR for the firm i at the time t given by:

$$\text{Co-CoVaR}_{mt}\left(1-\alpha^*\right) = \text{CoVaR}_{mt}\left(1-\alpha\right) + q_{it}^*,$$

with q_{it}^* the optimal adjustment for the firm i at the time t.

A second adjusted systemic risk measure is the Co-MES that corresponds to the partial derivative of the system adjusted Expected Shortfall with respect to the weight of a firm i in the economy such as:

$$\text{Co-MES}_{it}(1-\alpha) = \frac{\partial ES_{mt}^*(1-\alpha)}{\partial w_{it}} = E\left[r_{it}\left|r_{mt} \leq VaR_{mt}^*(1-\alpha)\right.\right], \tag{30}$$

where $ES_{mt}^*(\cdot)$ is the adjusted Expected Shortfall of the market m at the time t:

$$ES_{mt}^*(1-\alpha) = E\left[r_{mt}\left|r_{mt} \leq VaR_{mt}^*(1-\alpha)\right.\right],$$

and $VaR^*_{mt}(\cdot)$ is adjusted Value-at-Risk of the market m at the time t given by:

$$VaR^*_{mt}(1-\alpha) = VaR_{mt}(1-\alpha) + q^*_{mt},$$

with q^*_{mt} the optimal adjustment for the market m at the time t.

The Co-CES is an adjusted measure of CES that gives the absolute contribution of a firm to the risk of the financial system. It is obtained by calibrating the first derivative of the adjusted ES using weights w_{it} defined for each financial institution, such as (with the previous notations):

$$\text{Co-CES}_{it}(1-\alpha) = w_{it}E\left[r_{it}\left|r_{mt} \leq VaR^*_{mt}(1-\alpha)\right.\right]. \quad (31)$$

The Co-SRISK is an adjusted measure of SRISK that corresponds to the expected capital shortfall of a given financial institution, conditional on a crisis affecting the whole financial system such as (with the previous notations):

$$\text{Co-SRISK}_{it}(1-\alpha) = \max\left\{0, \gamma D_{it} - (1-\gamma)\, w_{it}\left[1 - LRMES^*_{it}(1-\alpha)\right]\right\}, \quad (32)$$

where $LRMES^*_{it}(1-\alpha)$ corresponds to the adjusted Long-Run Marginal Expected Shortfall, which is the expected equity loss of the firm i when market falls below 40% over six months:

$$LRMES^*_{it}(1-\alpha) \simeq 1 - \exp\left[18 \times MES^*_{it}(1-\alpha)\right].$$

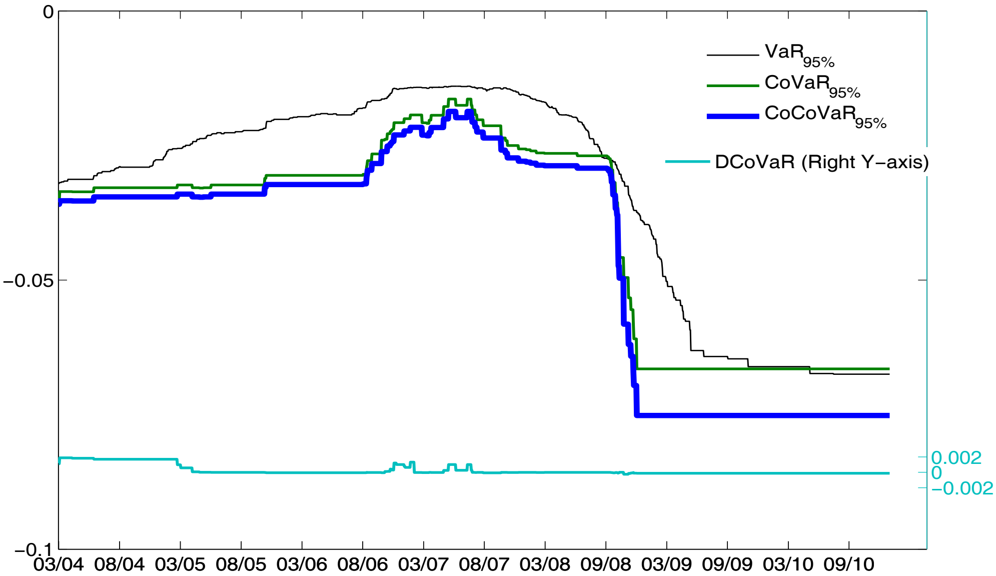
6.2. Empirical Evidences of Adjusted Systemic Risk Measures (To be Completed)

First, we illustrate our model risk correction procedure on the CoVaR measure for "Bank of America". Then we investigate how the ranking of the financial institution depends on the systemic risk measure considered and how this ranking is stabilized when implementing our correction. Finally, we control that the systemic risk measures as well as their corrected version are sensitive to a set of macroeconomic and financial condition variables.

Figure 8 presents the evolution of the following risk measures: VaR 95%, CoVaR 95% computed conditionally to individual VaR 90%, Co-CoVaR 95% that corresponds to corrected CoVaR 95% and the difference between VaR 99.50% and CoVaR 95% for Bank of America. This Figure first shows that systemic risk evaluated by CoVaR conditionally to a specific individual VaR of the studied asset can be seen as a *proxy* of a VaR defined for a higher threshold, since we can see in Figure 1 that the difference between VaR 99.50% and CoVaR 95% is very close to zero. Moreover, the CoVaR 95% that is more severe than VaR 95% is not validated by the Hit ratio test, Co-CoVaR 95% being in general inferior (more severe) than CoVaR 95%.

Those results are confirmed by the risk maps (see Colletaz *et al.*, 2012) realized on VaR and CoVaR with or without correction (see Figure 9). This Figure presents results obtained from out-of-sample estimates of a given risk model on a 4-year window (1,040 daily returns).

Figure 8: Comparison of VaR, CoVaR, and Co-CoVaR for Bank of America

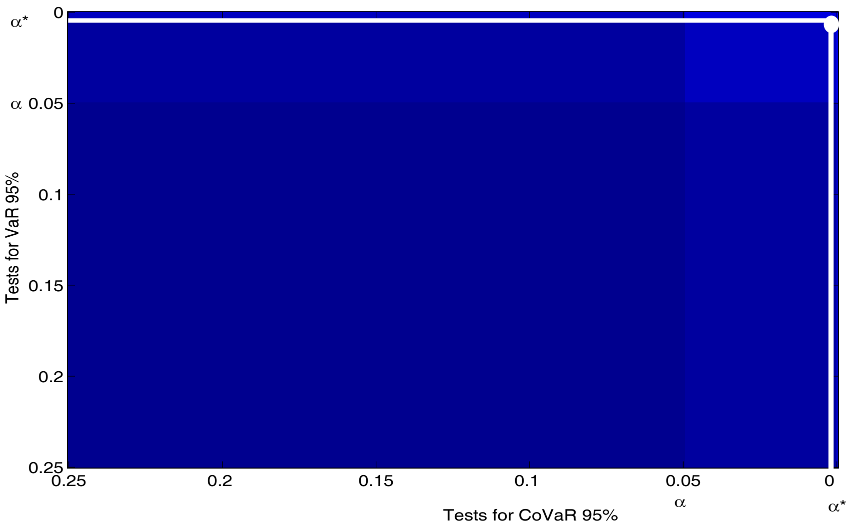


Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2012; computations by the authors. The figure presents the evolution of VaR 95%, CoVaR 95% computed conditionally to individual VaR 90%, Co-CoVaR 95% that corresponds to corrected CoVaR 95% and the difference between VaR 99.50% and CoVaR 95% (named DCoVaR) for Bank of America.

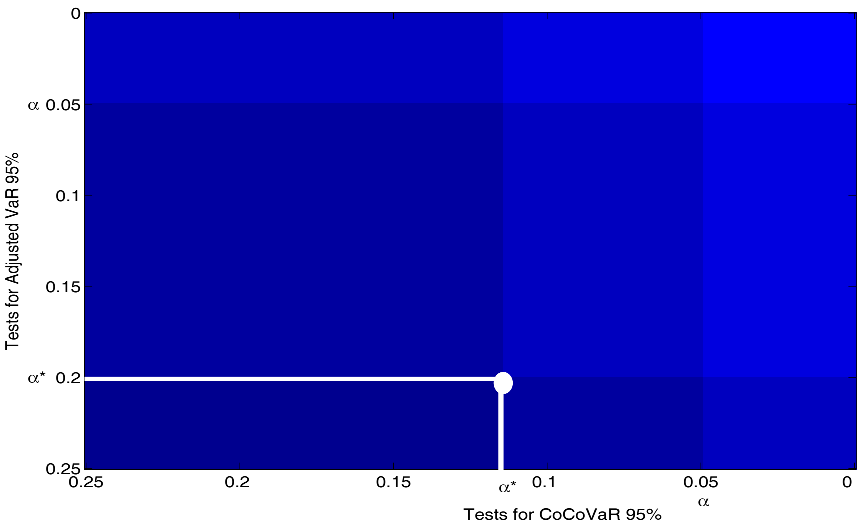
We thus dynamically estimate the parameters for the different methods. The Figure provides the risk map for VaR 95% *versus* CoVaR 95% (panel A) and for adjusted VaR 95% *versus* Co-CoVaR 95% (panel B). The x-axis and y-axis represent the critical probability (*p-value*, α^*) for the 95%-Hit ratio test realized on the risk measure for a threshold equals to 5%. The parameter α represents the significance threshold of the test; if the *p-value* is inferior to this value then we reject the hypothesis of the validity of the underlying model. The white circle in the Figure corresponds to the realized value of the test for both measures. We see that the critical values for both risk measures without corrections are inferior to significant threshold equals to 5.00% (white circle on the Panel A of the Figure 9). However, adjusted measures (adjusted VaR and Co-CoVaR, Panel B of Figure 9) are validated when using the same test for a threshold set to 5.00%. This result shows that we have to take into account model risk when estimating systemic risk through CoVaR.

Figure 9: Comparison of Map Risks with and without Correction for Bank of America

Panel A: Map Risk without Correction (VaR 95% versus CoVaR 95%)



Panel B: Map Risk with Correction (adjusted VaR 95% versus CoCoVaR 95%)



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors. This Figure represents the results of backests for a given risk model. We use a 4-year window dressing (1,040 daily returns) in order to dynamically estimate the parameters for the different methods. This Figure provides the map risks for VaR 95% versus CoVaR 95% (Panel A) and for adjusted VaR 95% versus Co-CoVaR 95% (Panel B). The x-axis and y-axis represent the p-value of the 95% Hit ratio test on the considered risk measure for a threshold equals to 5%. The parameter α represents the p-value of the significance test; if the p-value is less than this value thus we reject the validity hypothesis of the model. The white circle corresponds to the realized value of the test. In the Panel A (with no correction), we see that the p-values of the test are inferior to the significance test whereas they are superior in the Panel B (with correction).

Table 1: Rankings comparing Ten Financial Institutions characterized by the Biggest Impact on systemic risk according to VaR, CoVaR, CES, MES and SRISK

Rank	**VaR**			**CoVaR**			**CES**			**MES**			**SRISK**		
	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code
1	1	1	CIT	58	1	WU	59	13	JPM	94	1	ABK	49	13	BAC
2	2	1	ABK	10	1	NYX	71	13	C	93	1	ETFC	59	13	JPM
3	3	1	ETFC	7	1	ICE	49	13	BAC	67	1	LNC	71	13	C
4	4	1	CBG	12	1	AMP	34	11	WFC	81	1	MS	34	11	WFC
5	5	1	GNW	24	1	FNF	47	10	GS	71	1	C	47	10	GS
6	6	2	MBI	4	1	CBG	81	10	MS	90	1	MBI	81	10	MS
7	7	2	ICE	73	1	AIZ	58	6	AXP	78	2	AIG	78	9	AIG
8	8	3	AMTD	1	2	CITa	78	9	AIG	83	2	COF	63	8	HIG
9	9	3	JNS	5	2	GNW	33	5	USB	63	2	HIG	32	8	PNC
10	10	3	NYX	44	2	CME	48	7	BK	49	3	BAC	48	7	BK

Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors. This Table presents the ranking as of 31^{st} December, 2010 of Ten financial institutions having the biggest impacts on systemic risk as measured by VaR, CoVaR, MES, CES and SRISK. For each of these Systemic Risk Measures, rank in VaR in first columns are presented, RSOM clusters in the second columns, and names of financial institutions are displayed in the third columns (line by line, per decreasing systematic importance for the related risk measures - see appendix for codes of financial institutions). Clusters are here obtained grouping the various Systemic Risk Measures time-series according to a Robust Self-Organized Maps technique (see Guinot *et al.*, 2006; Sorjamaa *et al.*, 2009; Merlin *et al.*, 2010), first defining a string with 13 clusters ([1x13] aligned code vectors as a map), *i.e.* a number of classes close to the one used in Hurlin *et al.*, (2012).

Table 1 provides a ranking comparing the ten most important financial institutions according to several systemic importance measures: VaR, CoVaR, CES, MES and SRISK. This table highlights the heterogeneity and the variability of the different systemic risk measures in terms of SIFI rankings.

Clusters of systemic risk measures are also presented in this Table. More than on the figures themselves, we might indeed be interested in knowing whether a company is in a group of firms attached with a high systemic risk or not, and whether the Systemic Risk Measures of two financial institutions are different or similar. This is the approach adopted by Hurlin *et al.* (2012), who study groups of firms according to a bootstrap procedure inspired by the methodology of the Model Confidence Set of Hansen *et al.* (2011). They obtain this way clusters of equally systemically risky firms, in the sense that we cannot statistically distinguish two firms in the same cluster on the basis of their riskiness measure.

We use here, for building clusters of systemic risk measure time-series, a non-linear unsupervized Self-organized Map in its robust version (RSOM - see Guinot *et al.*, 2006; Sorjamaa *et al.*, 2009; Merlin *et al.*, 2010).

Hurlin *et al.* (2012) observe large differences in the very top of their ranking of firms based on MES, whilst it happens that contributions to systemic risk in the middle of the field are be very small and comparable, which leads them to conclude that it is unlikely that exists a relevant absolute ranking of systemic riskiness.

Table 2 provides the same rankings of SIFI according several model risk robust measures of systemic risk: adjusted VaR, Co-CoVaR, Co-CES, Co-MES and Co-SRISK. This table shows that all the classifications are similar for all adjusted systemic risk measures, very close to the unadjusted VaR original ranking. This result confirms that differences between the several systemic risk measures are not of the same magnitude than the model risk that should be considered, and thus are scratched when they are corrected for risk model. The suggested adjustment for risk model also makes it possible to improve the stability of measurements of systemic risks.

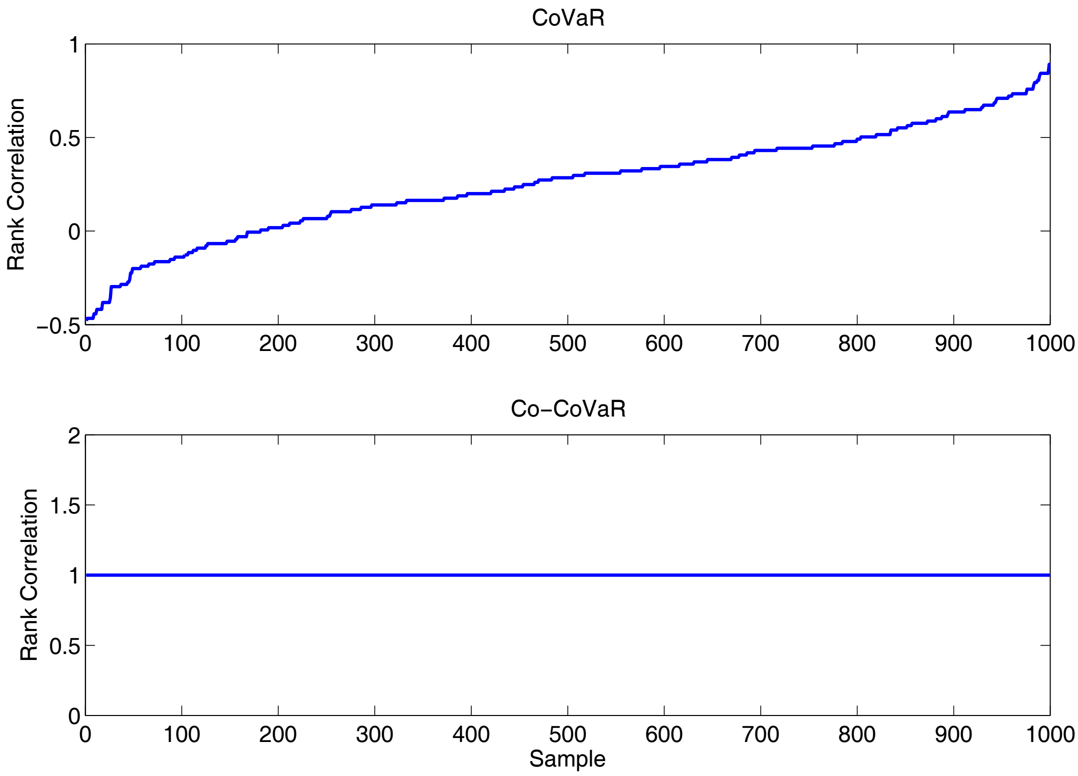
Figure 10 provides an ordered series of Spearman rank correlations for CoVaR and Co-CoVaR when considering some random noise on the VaR. More precisely, we generate 1,000 random series of noises with a null mean and variance equals to 50% of the cross-variance of VaR of financial institutions under studies over the whole sample. We add these series onto the original VaR, and, accordingly, recompute the related CoVaR and Co-CoVaR. The first part of the figure gives the rank correlation of the first ten financial institutions of the CoVaR and their corresponding rank on the CoVaR when shocks are added. The second part of the figure gives the rank of correlation of the first ten financial institutions of the Co-CoVaR and their corresponding rank on the Co-CoVaR still when the same shocks are applied to the original VaR series. Within sight of the figures, we note that our proposed corrected measure of systemic risk is very stable and robust to noise in the VaR measure.

Table 2: Ranking comparing Ten Financial Institutions characterized by the Biggest Impact on systemic risk according to adjusted VaR, Co-CoVaR, Co-CES, Co-MES, et Co-SRISK

	Adj. VaR			**Co-CoVaR**			**Co-CES**			**Co-MES**			**Co-SRISK**		
Rank	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code	Rank VaR	Clust. RSOM	Code
1	2	1	ABK	58	1	ABK	NA	NA	NA	NA	NA	NA	NA	NA	NA
2	3	1	ETFC	7	1	ETFC	NA	NA	NA	NA	NA	NA	NA	NA	NA
3	6	1	MBI	10	1	MBI	NA	NA	NA	NA	NA	NA	NA	NA	NA
4	1	1	CIT	12	1	CIT	NA	NA	NA	NA	NA	NA	NA	NA	NA
5	8	2	AMTD	24	2	AMTD	NA	NA	NA	NA	NA	NA	NA	NA	NA
6	9	2	JNS	1	2	JNS	NA	NA	NA	NA	NA	NA	NA	NA	NA
7	11	2	ACAS	4	2	ACAS	NA	NA	NA	NA	NA	NA	NA	NA	NA
8	13	2	COF	5	2	COF	NA	NA	NA	NA	NA	NA	NA	NA	NA
9	14	3	HBAN	73	3	HBAN	NA	NA	NA	NA	NA	NA	NA	NA	NA
10	15	3	MS	30	3	MS	NA	NA	NA	NA	NA	NA	NA	NA	NA

Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors. This Table presents the ranking as of 31^{st} December, 2010 of Ten financial institutions having the biggest impacts on systemic risk as measured by Adjusted VaR (*i.e.* corrected according to its efficiency), Co-CoVaR, Co-MES, Co-CES and Co-SRISK. For each of these Systemic Risk Measures, rank in terms of raw (non-adjusted) VaR are presented in first columns, RSOM clusters in the second columns, and names of financial institutions are displayed in the third columns (line by line, per decreasing systematic importance for the related corrected risk measures see appendix for codes of financial institutions). Clusters are here obtained grouping the various Systemic Risk Measure time-series according to a Robust Self-Organized Maps technique (see Guinot *et al.*, 2006; Sorjamaa *et al.*, 2009; Merlin *et al.*, 2010), first defining a string with 13 clusters ([1x13] aligned code vectors as a map), *i.e.* a number of classes close to the one used in Hurlin *et al.*, (2012).

Figure 10: Ordered Spearman Rank Correlation for CoVaR and Co-CoVaR of the Ten Financial Institutions on sub-samples with VaR Error



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors. This table gives the Spearman rank correlation for CoVaR and Co-CoVaR of the ten financial institutions with VaR Error. We generate for that, 1,000 random series with mean 0 and variance equal to 50% of the cross-variance of VaR for institutions. We add theses series on the VaR and compute the CoVaR and the Co-CoVaR. The first part of the figure gives the coefficient of correlation for the first ten financial institutions of the CoVaR with their corresponding rank on the CoVaR with shocks. The second part of the figure gives the coefficient of correlation of the Co-CoVaR with their corresponding rank on the Co-CoVaR with shocks. The x-axis is ranking following the level correlation coefficient.

For the sake of robustness, we hereafter provide some linear regression results regarding systemic risk measures (the dependent variables) and a set of macro-economic and financial conditions indicators (as explanatory variables), in order to assess the difference in the factor that governs the dynamic of the various systemic risk measures. We consider a set of explanatory variables that captures macroeconomic conditions and health of financial intermediaries (the states of nature): the implied volatility of the S&P equity index, VIX_t, the US economic surprise index (macroeconomic news), $NEWS_t$, the upward/downward earning revisions ratio as an indicator of the momentum of earnings expected by financial analysts, UD_t, the 3-month interbank rate, $LIBOR_t$, an interbank spread (the Ted spread), TED_t, the slope of the yield curve defines as the difference between the 10-year Treasury note and the 3-month Treasury bill rates, $TERM_t$, the market's return, Rm_t, the default spread defined as the yield differential between Moody's Aaa-rated and B-aa rated corporate bonds, DEF_t, an indicator of risk appetite defined as the rank correlation (Spearman correlation coefficient) between risks and (orthogonalized) returns of key global sectors, RA_t, and finally an indicator of liquidity measured by the growth of the net repurchase agreements (published by the Fed), LIQ_t.

We study hereafter the main forces driving the main systemic risk measures as well as their corrected version. We consider both time-series regression for each of the stocks in the sample and cross-section regressions between individual stocks for each of the 1723 days in the sample. Tables 3 to 6 report the average, minimum, maximum and standard deviations of the R^2 associated to the 1723 or 95 regressions, respectively. The sample period covers the 3^{rd} January, 2000 to the 31^{st} December, 2010.

In the Tables 3 and 4, we consider for each systemic risk measure, without and with correction for model risk respectively, a single-factor model in which the measure is successively explained by the implied volatility of the S&P500 index (VIX), Economic surprises index for the G10 (published by Citigroup), the up-down ratio of earning revisions by the financial analysts, the 3-month interbank interest rate (LIBOR), the Ted spread (3-month Libor minus 3-month T-Bill), the Term spread (10-year minus 3-month US Treasury securities), the default spread (BAA minus AAA corporate interest rates), the liquidity conditions measured by the growth of the net repurchase agreements and finally the VaR 90% of the market index.

In the Tables 5 and 6, we consider, without and with correction for model risk respectively, for each systemic risk measure a single-factor model in which the measure is successively explained by the stock return, the market capitalization, the book value, the book-to-market ratio, the beta and gamma ($= \rho^2\beta$, where ρ is the correlation coefficient between stock return and market return), and β the firm's beta. In the time-series dimension, an important part of the variance of the systemic risk measures are explained by the VaR of the market (from 25% for the SRISK to 53% for the CoVaR) and by the interbank interest rate (from 35% for the SRISK to 65% for the CoVaR). These results are globally the same with the adjusted systemic risk measures. It indicates that, even corrected, the systemic risk measures fluctuate with the aggregate financial and banking conditions.

Table 3: Explaining Systemic Risk Measures by Aggregate Risk Factors

		VIX	News	UD	Tx 3	ted	TERM	DEF	LIQ	VaRm90
VaR95	**Mean**	**0.13**	**0.01**	**0.04**	**0.70**	**0.12**	**0.54**	**0.21**	**0.10**	**0.63**
	Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.01
	Max	0.28	0.03	0.31	0.93	0.44	0.83	0.42	0.20	0.95
	std	0.08	0.01	0.07	0.21	0.09	0.17	0.08	0.05	0.21
CoVaR95	**Mean**	**0.24**	**0.00**	**0.03**	**0.65**	**0.04**	**0.51**	**0.19**	**0.17**	**0.53**
	Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.09
	Max	0.37	0.02	0.37	0.83	0.36	0.72	0.27	0.23	0.86
	std	0.08	0.00	0.05	0.20	0.04	0.15	0.07	0.06	0.11
DeltaCoVaR95	**Mean**	**0.33**	**0.01**	**0.06**	**0.54**	**0.01**	**0.42**	**0.17**	**0.22**	**0.32**
	Min	0.00	0.00	0.00	0.01	0.00	0.00	0.00	0.00	0.00
	Max	0.49	0.12	0.28	0.70	0.37	0.57	0.25	0.32	0.49
	std	0.13	0.02	0.04	0.19	0.04	0.15	0.05	0.09	0.11
MES95	**Mean**	**0.22**	**0.03**	**0.08**	**0.63**	**0.12**	**0.45**	**0.19**	**0.15**	**0.51**
	Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Max	0.75	0.59	0.64	0.92	0.85	0.86	0.43	0.55	0.93
	std	0.14	0.09	0.15	0.23	0.17	0.20	0.10	0.10	0.23
CES95	**Mean**	**0.22**	**0.02**	**0.11**	**0.47**	**0.09**	**0.34**	**0.17**	**0.14**	**0.40**
	Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Max	0.70	0.19	0.75	0.89	0.66	0.74	0.53	0.36	0.91
	std	0.16	0.03	0.16	0.24	0.14	0.20	0.13	0.10	0.23
SRISK95	**Mean**	**0.24**	**0.01**	**0.14**	**0.35**	**0.10**	**0.30**	**0.10**	**0.14**	**0.25**
	Min	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	Max	0.68	0.12	0.61	0.80	0.54	0.74	0.29	0.41	0.85
	std	0.17	0.02	0.14	0.25	0.12	0.21	0.08	0.10	0.22

Source: Datastream and Bloomberg, daily data in USD from the 2^{nd} March, 2004 to the 31^{st}, December, 2010; computations by the authors. This table presents some R^2 statistics (average [in bold face], minimum, maximum, and standard deviation) obtained by regressing a systemic risk measure on one aggregate risk factor. Abbreviations stand for: the implied volatility of the S&P equity index, VIX, the US economic surprise index (macroeconomic news), NEWS, the upward/downward earning revisions ratio as an indicator of the momentum of earnings expected by financial analysts, UD, the 3-month interbank rate, LIBOR, an interbank spread (the Ted spread), TED, the slope of the yield curve defines as the difference between the 10-year Treasury note and the 3-month Treasury bill rates, TERM, the default spread defined as the yield differential between Moody's Aaa-rated and B-aa rated corporate bonds, DEF, an indicator of risk appetite defined as the rank correlation (Spearman correlation coefficient) between risks and (orthogonalized) returns of key global sectors, RA,an indicator of liquidity measured by the growth of the net repurchase agreements (published by the Fed), LIQ, and finally the 90% market's Value-at-Risk, VaRm90.

Table 4: Explaining Adjusted Systemic Risk Measures by Aggregate Risk Factors

		VIX	News	UD	Tx 3	ted	TERM	DEF	LIQ	VaRm90
VaR95	**Mean**	**0,13**	**0,01**	**0,04**	**0,70**	**0,12**	**0,54**	**0,21**	**0,10**	**0,62**
	Min	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,00	0,01
	Max	0,29	0,03	0,31	0,93	0,44	0,82	0,42	0,20	0,95
	std	0,08	0,01	0,07	0,21	0,09	0,16	0,08	0,05	0,21
CoVaR95	**Mean**	**0,24**	**0,00**	**0,03**	**0,66**	**0,04**	**0,51**	**0,19**	**0,18**	**0,52**
	Min	0,00	0,00	0,00	0,08	0,00	0,06	0,00	0,00	0,13
	Max	0,43	0,02	0,36	0,83	0,43	0,72	0,27	0,29	0,86
	std	0,09	0,00	0,05	0,18	0,05	0,13	0,07	0,07	0,11
DeltaCoVaR95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA	NA	NA	NA
MES95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA	NA	NA	NA
CES95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA	NA	NA	NA
SRISK95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA	NA	NA	NA

Source: Datastream and Bloomberg, daily data in USD from the 2^{nd} March, 2004 to the 31^{st}, December, 2010; computations by the authors. This table presents some R^2 statistics (average [in bold face], minimum, maximum, and standard deviation) obtained by regressing adjusted systemic risk measure on one aggregate risk factor. Abbreviations stand for: the implied volatility of the S&P equity index, VIX, the US economic surprise index (macroeconomic news), NEWS, the upward/downward earning revisions ratio as an indicator of the momentum of earnings expected by financial analysts, UD, the 3-month interbank rate, LIBOR, an interbank spread (the Ted spread), TED, the slope of the yield curve defines as the difference between the 10-year Treasury note and the 3-month Treasury bill rates, TERM, the default spread defined as the yield differential between Moody's Aaa-rated and B-aa rated corporate bonds, DEF, an indicator of risk appetite defined as the rank correlation (Spearman correlation coefficient) between risks and (orthogonalized) returns of key global sectors, RA,an indicator of liquidity measured by the growth of the net repurchase agreements (published by the Fed), LIQ, and finally the 90% market's Value-at-Risk, VaRm90.

Table 5: Explaining Systemic Risk Measures by Firm Characteristics

		R_i	MV	BV	B/M	beta	gamma
VaR95	**Mean**	**0.07**	**0.01**	**0.00**	**0.10**	**0.58**	**0.12**
	Min	-0.01	-0.01	-0.01	-0.01	-0.01	0.01
	Max	0.67	0.09	0.02	0.40	0.73	0.44
	std	0.10	0.03	0.01	0.13	0.08	0.13
CoVaR95	**Mean**	**0.02**	**0.01**	**0.00**	**0.00**	**0.03**	**0.18**
	Min	-0.01	-0.01	-0.01	-0.01	-0.01	0.01
	Max	0.29	0.05	0.05	0.08	0.35	0.39
	std	0.04	0.02	0.01	0.02	0.05	0.11
DeltaCoVaR95	**Mean**	**0.01**	**0.01**	**0.01**	**0.00**	**0.03**	**0.08**
	Min	-0.01	-0.01	-0.01	-0.01	-0.01	-0.01
	Max	0.33	0.06	0.07	0.12	0.23	0.34
	std	0.04	0.02	0.02	0.02	0.04	0.10
MES95	**Mean**	**0.07**	**0.00**	**0.03**	**0.13**	**0.92**	**0.54**
	Min	-0.01	-0.01	0.00	-0.01	0.86	0.24
	Max	0.62	0.05	0.07	0.54	0.95	0.87
	std	0.10	0.01	0.02	0.14	0.02	0.21
CES95	**Mean**	**0.00**	**0.70**	**0.57**	**0.02**	**0.04**	**0.12**
	Min	-0.01	0.56	0.35	-0.01	-0.01	0.02
	Max	0.20	0.90	0.72	0.12	0.11	0.22
	std	0.02	0.06	0.07	0.04	0.04	0.06
SRISK95	**Mean**	**0.00**	**0.82**	**0.94**	**0.12**	**0.00**	**0.06**
	Min	-0.01	0.52	0.87	-0.01	-0.01	0.02
	Max	0.25	0.90	0.98	0.27	0.03	0.14
	std	0.03	0.07	0.03	0.09	0.01	0.03

Source: Datastream and Bloomberg, daily data in USD from the 2^{nd} March, 2004 to the 31^{st}, December, 2010; computations by the authors. This table presents some R^2 statistics (average [in bold face], minimum, maximum, and standard deviation) obtained by regressing a systemic risk measure on one cross-sectional risk factor. Abbreviations stand for: the stock return, R_i, the market capitalization, MV, the book value, BV, the book-to-market ratio, B/M, the beta and gamma $= \rho^2\beta$, where ρ is the correlation coefficient between stock return and market return, and β the firm's beta.

Table 6: Explaining Systemic Risk Measures by Aggregate Risk Factors

		R_i	MV	BV	B/M	beta	gamma
VaR95	**Mean**	**0,07**	**0,01**	**0,00**	**0,11**	**0,58**	**0,12**
	Min	-0,01	-0,01	-0,01	-0,01	-0,01	0,00
	Max	0,68	0,09	0,02	0,40	0,72	0,43
	std	0,10	0,03	0,01	0,13	0,08	0,12
CoVaR95	**Mean**	**0,01**	**0,00**	**0,00**	**0,01**	**0,02**	**0,16**
	Min	-0,01	-0,01	-0,01	-0,01	-0,01	-0,01
	Max	0,35	0,06	0,04	0,06	0,29	0,39
	std	0,04	0,01	0,01	0,01	0,05	0,11
DeltaCoVaR95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA
MES95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA
CES95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA
SRISK95	**Mean**	**NA**	**NA**	**NA**	**NA**	**NA**	**NA**
	Min	NA	NA	NA	NA	NA	NA
	Max	NA	NA	NA	NA	NA	NA
	std	NA	NA	NA	NA	NA	NA

Source: Datastream and Bloomberg, daily data in USD from the 2^{nd} March, 2004 to the 31^{st}, December, 2010; computations by the authors. This table presents some R^2 statistics (average [in bold face], minimum, maximum, and standard deviation) obtained by regressing a systemic risk measure on one cross-sectional risk factor. Abbreviations stand for: the stock return, R_i, the market capitalization, MV, the book value, BV, the book-to-market ratio, B/M, the beta and gamma $= \rho^2\beta$, where ρ is the correlation coefficient between stock return and market return, and β the firm's beta.

In the cross-sectional dimension, 92% of the variance of the MES is explained by the beta but this part decreases to 75% when considering the adjusted MES. At the same time, 94% of the variance of the SRISK is explained by the book value (and 82% by the market value). These results are qualitatively the same with the adjusted versions of the systemic risk measures.

Overall, these results confirm findings by Benoit *et al.* (2013) that the SRISK captures the size dimension while the MES captures the market risk (beta). Moreover, the adjustment procedure for model risk does not disturb these properties.

7. Conclusion

The trend toward financial globalization in the past decades has strengthened the interconnexions between markets and financial institutions. As a consequence, both the potential efficiency and the risk of disruptions of the financial system have increased. Properly measure the systemic importance of financial institutions and identify SIFI is a crucial but difficult task when managing the systemic risk. Several authors (Gouriéroux and Monfort, 2013; Acharya *et al.*, 2010) have already shown that the systemic risk measure matters and that the related chosen tax system is not neutral.

In the vein of Daníelsson *et al.* (2011) and Benoit *et al.* (2013), the goal of this article is first to propose a comparison of the major systemic risk measures (CoVaR, ΔCoVaR, MES, CES, SRISK), when, this time, model risk is considered. Indeed, numerous research have highlighted the importance of such a risk in VaR computation.

We show that 1) the ranking in terms of systemic risk impact of financial institutions highly depend on the adopted measure, and 2) , since they heavily rely on quantile estimates, they are very sensitive of errors in the measure of extreme quantiles and might be largely impacted by mild mis-measurements of extreme risk of financial companies. In other words, we may say that 1) it might be vain to try to assess which measure is best (according to which decision *criterion*?); 2) related systemic risk rankings might be largely unfair for some financial institutions since a mere random luck may affect them, and 3) the aim of stabilization of the system might not reachable with current systemic measures, due to (certain) potential errors in the rankings of SIFI, and may have some competition distortion effects: regulator strong supervision scrutinized brand name argument *versus* mere fears of clients to be in the hands of a spot-lighted SIFI.

We then use the approach of Boucher *et al.* (2013) in order to suggest a simple first correction that stabilizes systemic rankings, avoiding them to be merely arbitrary. We thus start studying the case of the CoVaR (Adrian et Brunnermeier, 2011), and its corrected version (called Co-CoVaR), before generalizing our approach to other main systemic risk measures such as MES, CES and SRISK. We finally show that suggested corrections are of importance and that the highlighted systemic institutions are different from those identified when using the standard non-corrected risk measures. Furthermore, it happens that, when corrected for model risk, all tested measures lead to the same ranking of SIFI, which appears to be the one of the adjusted VaR for model risk, which is very close to the one of raw uncorrected VaR of institutions.

The next step will be to further analyze the factors explaining the systemic measures in their various original and corrected versions (static mono-factor as in Brownlees and Engle, 2012 and Benoit *et al.*, 2013, *versus* multi-factorial analyses with dynamic betas as in Engle *et al.*, 2012)

Finally, even if at this stage further investigations must be conducted and if precautions should be advocated, we may conclude at the end that model risk in the computation of risk measures should really be considered for the sake of the fairness of the system and the global financial system stability.

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Appendix A. Dataset Description

Table A.7: Company Names and Industry Groups

Code	Retail Banks	Code	Retail Banks
BAC	Bank of America	BBT	BB&T
BK	Bank of New York Mellon	C	Citigroup
CBH	Commerce Bancorp	CMA	Comerica Inc.
HBAN	Huntington Bancshares	HCBK	Hudson City Bancorp
JPM	JP Morgan Chase	KEY	Keycorp
MI	Marshall & Ilsley	MTB	M&T Bank Corp.
NCC	National City Corp.	NTRS	Northern Trust
NYB	New York Community Bancorp	PBCT	Peoples United Financial
PNC	PNC Financial Services	RF	Regions Financial
SNV	Synovus Financial	SOV	Sovereign Bancorp
STI	Suntrust Banks	STT	State Street
UB	Unionbancal Corp.	USB	US Bancorp
WB	Wachovia	WFC	Wells Fargo & Co
WM	Washington Mutual	WU	Western Union
ZION	Zions		
Code	**Insurance**	**Code**	**Insurance**
ABK	Ambac Financial Group	AET	Aetna
AFL	AFLAC	AIG	American International Group
AIZ	Assurant	ALL	Allstate Corp.
AOC	Aon Corp.	BKLY	W.R. Berkley Corp.
BRK	Berkshire Hathaway	CB	Chubb Corp.
CFC	Countrywide Financial	CI	CIGNA Corp.
CINF	Cincinnati Financial Corp.	CNA	CNA Financial Corp.
CVH	Coventry Health Care	FNF	Fidelity National Financial
GNW	Genworth Financial	HIG	Hartford Financial Group
HNT	Health Net	HUM	Humana
LNC	Lincoln National	MBI	MBIA
MET	MetLife	MMC	Marsh & McLennan
PFG	Principal Financial Group	PGR	Progressive
PRU	Prudential Financial	SAF	Safeco
TMK	Torchmark	TRV	Travelers
UNH	UnitedHealth Group	UNM	Unum Group

Table A.8: Company Names and Industry Groups (Cn't)

Code	Brokers	Code	Brokers
AGE	A.G. Edwards	BSC	Bear Stearns
ETFC	E*Trade Financial	GS	Goldman Sachs
LEH	Lehman Brothers	MER	Merill Lynch
MS	Morgan Stanley	NMX	Nymex Holdings
SCHW	Schwab Charles	TROW	T. Rowe Price
Code	**Others**	**Code**	**Others**
ACAS	American Capital	AMP	Ameriprise Financial
AMTD	TD Ameritrade	AXP	American Express
BEN	Franklin Resources	BLK	BlackRock
BOT	CBOT Holdings	CBG	C.B. Richard Ellis Group
CBSS	Compass Bancshares	CIT	CIT Group
CME	CME Group	COF	Capital One Financial
EV	Eaton Vance	FITB	Fifth Third Bancorp
FNM	Fannie Mae	FRE	Freddie Mac
HRB	H&R Block	ICE	Intercontinental Exchange
JNS	Janus Capital	LM	Legg Mason
NYX	NYSE Euronext	SEIC	SEI Investment Company
SLM	SLM Corp.		

Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; database used in Benoit *et al.* (2013). This table presents all the american financial firms with a market capitalization superior to 5 dollar billions as of July 2007.

Appendix B. Sketch of Proofs of Propositions (work-in-progress)

Proof. of Proposition 1

First equation comes from Benoit *et al.* (2013) and we have (*per* definitions of the β_{it} and of the $ES_{mt}(\alpha)$):

$$\begin{aligned} MES_{it}(\alpha) &= \sigma_{it}\rho_{it}E_{t-1}\left(\varepsilon_{mt}\left|r_{mt} < VaR_{mt}(\alpha)\right.\right) \\ &= \beta_{it}\sigma_{mt}E_{t-1}\left(\varepsilon_{mt}\left|r_{mt} < VaR_{mt}(\alpha)\right.\right) \\ &= \beta_{it}E_{t-1}\left(r_{mt}\left|r_{mt} < VaR_{mt}(\alpha)\right.\right) \\ &= \beta_{it}ES_{mt}(\alpha). \end{aligned} \tag{B.1}$$

Second equation comes from Andreev *et al.* (2005) who show that:

$$ES_{it}(\alpha) = \frac{\nu + \left[VaR_{it}(\alpha)\right]^2}{\nu - 1}\left\{\frac{st\left[VaR_{it}(\alpha)\right]}{1-\alpha}\right\}. \tag{B.2}$$

The third equation comes from Boucher *et al.* (2013) and the fact that we can re-express, under some conditions, a transformed $VaR_{it}(\alpha)$ as a $VaR_{it}(\widetilde{\alpha}^*)$. ■

Proof. of Proposition 2
First equation comes from Benoit *et al.* (2013), see proposition 1, and the second from Boucher *et al.* (2013) since a transformed $VaR_{it}(\alpha)$ is a $VaR_{it}(\widetilde{\alpha}^*)$. ■

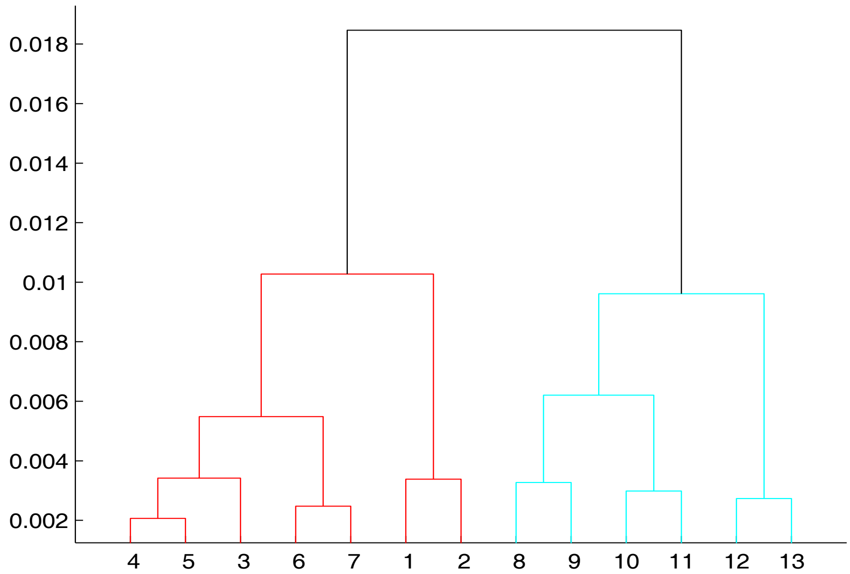
Proof. of Proposition 3
First equation comes from Benoit *et al.* (2013), the second is a direct application of previous Propositions 1 and 2, and the third from Boucher *et al.* (2013) since a transformed $VaR_{it}(\alpha)$ can be expressed as a $VaR_{it}(\widetilde{\alpha}^*)$. ■

Table B.9: Comparison of Classification for Financials Institutions based on MES Risk Measure (Hurlin *et al.*, 2012, as of 2008 and ROM Cluster as of 31^{st} December, 2010)

Tick	RSOM Cluster	Hurlin *et al.* (2012) Cluster	Tick	RSOM Cluster	Hurlin *et al.* (2012) Cluster	Tick	RSOM Cluster	Hurlin *et al.* (2012) Cluster
MBI	1	1	BBT	8	4	HIG	2	8
KEY	4	1	WFC	7	4	ACAS	3	8
RF	4	1	BEN	6	4	AFL	8	8
HBAN	5	1	LNC	1	4	HCBK	13	8
STI	5	1	AMTD	5	4	CNA	9	9
MS	1	2	SLM	7	4	CB	11	9
AXP	5	2	LM	3	4	AOC	13	9
FITB	3	2	BK	6	5	PBCT	13	9
COF	1	2	BAC	3	5	PGR	12	9
AIG	1	3	BLK	10	5	HRB	12	10
C	1	3	SCHW	4	5	TMK	10	10
TRV	11	3	MTB	10	5	NYB	12	11
MI	4	3	SEIC	8	5	HUM	11	11
ZION	6	3	STT	3	6	ALL	10	11
CMA	6	3	CINF	11	6	CI	9	11
TROW	4	3	GS	6	6	WRB	9	11
ETFC	1	3	PNC	8	7	UNH	13	11
EV	5	3	UNM	5	7	CVH	9	11
SNV	7	4	NTRS	7	7	HNT	13	12
JPM	3	4	USB	9	7	BRK	13	13

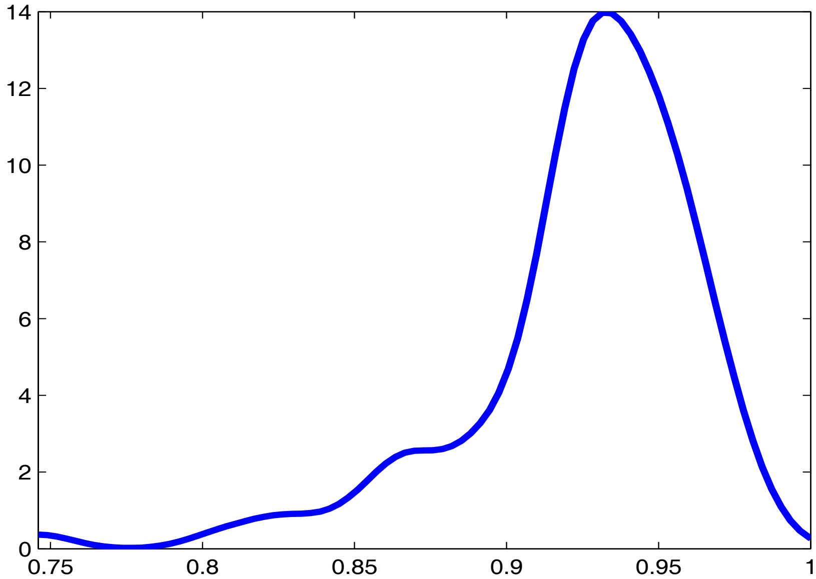
Source: *COMPUSTAT* and *CRSP*, daily data in USD from 3^{rd} January, 2000 to 31^{st} December, 2010; database used in Benoit *et al.* (2013). This table presents all the american financial firms with a market capitalization superior to 5 USD billions as of July 2007.

Figure B.11: Dendrogram of Classification with Robust Organized Map for Financials Institutions based on MES Risk Measure as of 31^{st} December, 2010)



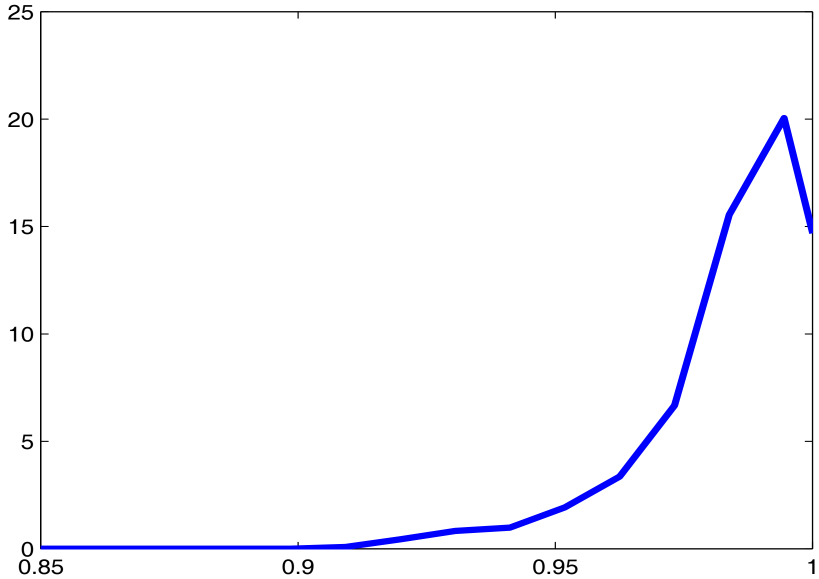
Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors.

Figure B.12: *Alphas* (denoted, $\widetilde{\alpha}_{it}^{***}$) Distribution of Financial Institutions VaR for Proposition 2



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors.

Figure B.13: *Alphas* ($\widetilde{\alpha}_{it}^{****}$) Distribution of Market VaR for Proposition 3



Source: *COMPUSTAT* and *CRSP*, daily data in USD from the 3^{rd} January, 2000 to the 31^{st} December, 2010; computations by the authors.